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# The indigenous culture of school mathematics in China and the United States: a comparative study of teachers' understanding of constructivism

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THE INDIGENOUS CULTURE OF SCHOOL MATHEMATICS IN CHINA AND THE  
UNITED STATES: A COMPARATIVE STUDY OF TEACHERS' UNDERSTANDING OF  
CONSTRUCTIVISM

A Dissertation

Submitted to the Graduate Faculty of the  
Louisiana State University and  
Agricultural and Mechanical College  
in partial fulfillment of the  
requirements for the degree of  
Doctor of Philosophy

in  
The Department of Educational Theory, Policy, and Practice

by  
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December, 2009

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## ABSTRACT

This study aimed to explore how the indigenous (national) culture of teaching and learning mediates teachers' understandings of constructivism in China and the U.S. Thirty middle school math teachers who are self-identified with the mathematics teaching reform movement in each country participated in this study (NCTM 2000 Math Standards in the United States or the MOE 2001 Math Standards in China).

Both theoretical and empirical methods were adopted for this research. Theoretical analysis led to a new cultural model that helped select appropriate cultural elements for this study. Based on emergence theory, the new model perceives Confucianism and Taoism as the most influential beliefs and values in terms of teaching and learning in China, in contrast with Behaviorism and Individualism in the U.S.

This study revealed that the indigenous culture of China and U.S. greatly influenced teachers' understandings of teaching and learning. Chinese participants tended to advocate Eastern belief that math learning develops through mental struggle, and is facilitated by providing hints, whereas their American counterparts tended to have faith in the Western belief that properly sequenced instruction supplemented by general encouragement of students will lead to learning. However, in some cases teachers' responses defied the predictions of the cultural model. For instance Chinese and American teachers both tended to opt for the Eastern belief of creating pedagogical balance as opposed to the Western belief in choosing a single well-chosen method. The differences and commonalities between Chinese and American participants' understandings of learning and teaching are thoroughly explored in this study. The key issue of transportability of recommended pedagogical practices across cultural boundaries is discussed in the Conclusions section.

# CHAPTER ONE: INTRODUCTION

## Introduction to Comparative Study

Although comparative education can be traced back to Greek times, there is no consensus on how to define the term. In the UK's tradition, the term "comparative" investigation was usually associated with the western developed countries while the term "international" education indicates the study was conducted in developing countries. Hall (1990) classified comparative education into four aspects: comparative studies, education abroad, international education, and development education (p. 23). Recently many researchers have adopted Halls' classification.

The term "comparative study" used in my dissertation indicates both comparative studies and international education. Since my attention is to the comparisons of mathematics education among nations, my foci are narrowed down to math curriculum and pedagogy in different countries.

During the last two decades, comparative study has attracted much attention from researchers. One factor stimulating comparative study is the large-scale international comparisons. Large-scale international mathematics comparisons reached their peak with the Trends in International Mathematics and Science Study (TIMSS) in 1999, and continue to increase the number of the participating countries. In 2003, there were fifty-two countries participating in TIMSS. In 2007, nearly seventy countries took part in the TIMSS investigations. Other influential large-scale studies such as International Assessment of Educational Progress (IAEP) and Programme for International Student Assessment (PISA) also have been conducted during this period.

Another factor stimulating comparative study is globalization (Arnone, & Torres, 2007; Barker, & Wiseman, 2007). Since the 1990s, globalization has become a central theme in

political, economic, and cultural debates (Alexander, 2000). Forced by the pressure of globalization, educational reforms are revamping educational standards to be more competitive. For instance, The U.S. published its updated standard and principles for school mathematics in 2000. In the same year, Japan started its new math curriculum. In 2001, China initiated a new standard for 1-9 school mathematics. In addition, South Korea, Singapore, Malaysia, and New Zealand also enacted their math curriculum reforms in the 1990s (Wong, Han, & Lee, 2004).

The aims of comparative study can be articulated in two phrases, “to understand” and “to learn from,” which also represent the general agreement on the purposes of comparative education among comparative researchers (e.g., Halls, 1990; Holms, 1971; Lauwerys, 1969; Phillips & Schweisfurth, 2006). During the last two decades, researchers in comparative study have conducted extensive investigations on the first goal “to understand”. Based on the TIMSS study, Asian countries such as China, South Korea, and Japan have been identified as good examples on the students’ achievement and the teaching practice. To understand how Chinese learn mathematics, Fan, Wong, Cai, and Li (2004) compiled recent comparative work from four dimensions: overview and international perspectives, context and teaching materials, pedagogy and learning processes, and inspiration and future directions. They have made great efforts on identifying Chinese learning and teaching as a good example for other countries.

The second goal, “to learn from,” might be the final goal for comparative study, as Phillips and Schweisfurth (2006) pointed out:

The most obvious consequence of learning from and understanding what is happening ‘elsewhere’ in education is that we might be persuaded of the advantages to be gained from copying or emulating successful practice as it is manifest in other countries—what has become generally known as ‘borrowing.’ (p. 17)

The second goal, however, seems ambiguous in the literature of comparative study. Most comparative researchers have been interested in designing their research by asking the following questions: 1) What is the nature of? 2) What is the situation of x in the context of y? 3) How

different/similar—in terms of x, is a from b in the context y? 4) Given that we can observe differences in terms of x, between a and b in the context of y, what might explain those differences/similarities? 5) What are the implications of such similarities/differences for the separate context of z? (Phillips & Schweisfurth, 2006, p. 99). The last question informs the research agenda “to learn from”. However, assimilating a good practice from the outside is extraordinarily complex. A living plant metaphor can help us understand this complexity:

We cannot wander at pleasure among the educational systems of the world, like a child strolling through a garden, and pick off a flower from one bush and some leaves from another, and then expect that if we stick what we have gathered into the soil at home, we shall have a living plant. A national system of education is a living thing, the outcome of forgotten struggles and difficulties, and ‘of battles long ago’. It has in it some of the secret working of national life. (Phillips & Schweisfurth, 2006, p. 18)

Policy makers tend to pick up “a flower” quickly from the outside and expect it to bloom at home. For instance, in the mid 1990s, the British government espoused “the simple nostrum that the key to enhanced standards and economic comparativeness was an unrelenting concentration on basic skills in literacy and numeracy, to be taught mainly through that ‘interactive whole-class teaching’ which was used in schools in Germany, Switzerland, Hungary, Taiwan and Japan” (Alexander, 2000, p. 2). In China, the similar reform happened in the late 1990s.

A few researchers (Thomas, 1997; Phuong, Terlouw, & Pilot, 2006) have noticed the above problems since the late 1990s. They did not believe that simply making a comparison of the differences and similarities among countries in comparative study is adequate to reach the goal “to learn from.” These researchers warned that the national culture played a fundamental role in the adoption of a promising practice from other cultures. Phuong, Terlouw, and Pilot (2006) examined the group learning in a Confucian Heritage Culture (CHC) context. They posed ten questions as relevant to the culture influence on group learning. For instance, they asked:

Question 7: In what way can CHC learners become independent and autonomous enough to carry on group learning activities according to the expectations of Western models?

Question 8: In what way can CHC learners surmount the preference for a structured style in order to reach the level of collaborative participation in group work? (p. 10)

The important contribution of Phuong, Terlouw, and Pilot's investigation is they are directly dealing with the *transportability* of the practice from one culture context to another culture context. That is, they identify the original culture first, and then analyze the possibility and viability of transferring a promising practice to other culture contexts.

In my dissertation, I am also concerned with transportability issues, between China and the U.S. Specifically, I am interested in how the indigenous culture influences teachers' understanding of the new mathematics standard and principles in each country. This study will help audiences better understand teachers' interpretations of reform documents in China and the USA, and articulate the connections between the culture of teaching and the indigenous national culture. The motivation of this study is to understand the transportability of theoretical perspectives and the transportability of teaching practices. More detailed description will be found in the next section.

## Overview and Organization of the Dissertation

### Overview of the Dissertation

In 2000, the National Council of Teachers of Mathematics (NCTM) published an update to its standards called *Principles and Standards for School Mathematics*. One year later, the Chinese Ministry of Education published a similar document entitled *Mathematics Curriculum Standards*, which became the basis of China's new mathematics reform in 2001. The Chinese standards were modeled closely after the U.S. example. In this math education reform movement, constructivism is often cited as one of the major theoretical underpinnings of reform. Indeed, constructivist teaching has become almost a synonym for reform teaching. In my dissertation, I am concerned with middle school reform teachers' understanding of constructivism as mediated by the indigenous culture of learning and teaching in their native country. I define the reform

teachers as those who presently teach math lessons based on the new standards mentioned above. The indigenous culture in my study will be interpreted in terms of fundamental beliefs regarding learning and teaching in both countries. For instance, the roots of Chinese indigenous culture are Confucianism and Taoism. The roots of the U.S. indigenous culture are behaviorism and individualism. Since little work has been done on this issue in math education, I will come up with a model of indigenous culture of teaching and learning in each country.

### Research Questions

Research question 1: What are U.S. and Chinese teachers' understandings of constructivism as embedded in the math standards documents from their countries?

This research question was addressed through a reform-orientation questionnaire that established the extent to which teachers identify themselves with the reform agenda; through a constructivism questionnaire that examined the teacher's nuanced interpretations of constructivism, and through an analysis of the US and Chinese reform documents to identify the embedded constructivist assumptions.

Research question 2: What aspects of U.S. indigenous culture and Chinese indigenous culture are relevant to learning and teaching?

This research question was addressed through traditional cultural contexts in china and the USA, through sociology theory (e.g., emergence theory), and through current cultural studies regarding teaching and learning in comparative education.

Research question 3: How does the indigenous culture of China and U.S. influence teachers' understandings of learning?

This research question was addressed through the interview and teaching episode data.

Research question 4: How does the indigenous culture of China and U.S. influence teachers' understandings of teaching? This research question was addressed through the interview and teaching episode data too.

## A Model of Influences on Teachers' Understanding of Constructivism

Figure 1.1 illustrates the model of cultural influences on teaching that underlie the study. Theorists' views of constructivism in box 1 are an influence on the standards documents in box 2. Box 3 serves as a mediator between box 2 and box 4. Box 3, box 4, and box 5 constitute a feedback loop reflecting the mutual influences and changes over time among the indigenous culture of teaching and learning, teachers' understandings of constructivism, and teachers' actual teaching practice.

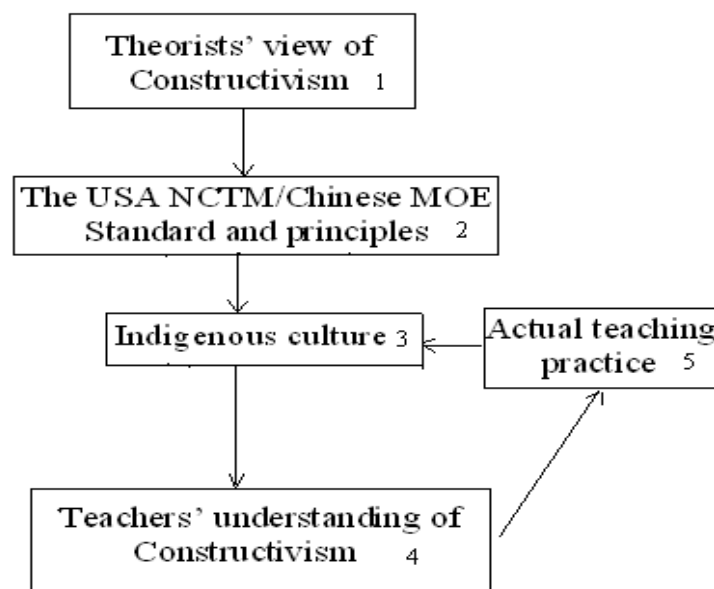


Figure 1.1: A Model of Influences on Teachers' Understanding of Constructivism

As shown in Figure 1.1, starting from the theorists' view of constructivism, I will analyze the elements of various constructivist theories reflected in the U.S. NCTM's and Chinese MOE's standards and principles. This analysis will assess the theoretical rigor and limitations of the two standards documents. I will also demonstrate that the two standards documents are based on very similar reform tenets. The indigenous culture of learning and teaching is not explicitly interpreted in either standards document. Thus, reform teachers learn similar tenets of teaching and learning based on the two standards in both countries. However, it is not my expectation that

teachers in both countries have the similar understandings of constructivism. In different culture contexts, teachers' interpretations of a theory will be influenced by the indigenous culture. The mutual influences among box 3, box 4, and box 5 will help audiences understand the transportability of teaching practices. The analysis of these influences also helps me reach the research goal "to learn from". That is, identify the indigenous culture in each county first, then analyze the mutual relationships among the indigenous culture, teachers' understanding of constructivism, and actual teaching practice, and then discuss the possibility and viability of transferring a promising practice to the other culture contexts.

### Research Design

The detailed descriptions of the methodology of my dissertation will be found in Chapter Three. The research was designed as a comparative case study. Thirty middle school teachers in each country were selected in this study, teachers who use the reform math approach in their classes. My study focused on math educational reform during the period of 2000 to 2008.

Purposeful sampling (Creswell, 2005, p. 204) were used in this study. In China, thirty middle school math teachers were selected from the Changchun area including Changchun city and its five counties. I was assisted in finding subjects and collecting data in China by the contact person, Zhang Anli. American counterparts were selected from the Baton Rouge area including East Baton Rouge Parish and West Baton Rouge Parish.

The Constructivism-Culture and Actual Teaching Practice Survey (CCATPS) was developed and used in this research. This survey helped me gain a picture of the teachers' understandings of constructivism and reform teaching practices. In order to obtain rich and in-depth information for data analysis, the CCATPS contains four parts. Part I is a reform-orientation questionnaire; part II is a teaching-style questionnaire; part III is a values questionnaire; and part IV is the free writing of teachers about their actual teaching practice.



All thirty subjects in each country were asked to complete the survey. Six subjects in each country were selected for a one-hour interview that follow-up and extend the responses in the questionnaire. Telephone interviews were used with Chinese subjects, while face-to-face interviews were used with the USA subjects. An interview protocol was developed before I started the interviews. All interview data were audio-taped. Although the data collection methods (telephone and face-to-face interview) are different for the two countries, this difference will help compensate for the fact that the Chinese interviews are conducted in my native language.

### Organization of the Dissertation

The dissertation comprises five chapters. Chapter One is the introduction. The historical background of math education reform and the overview of the dissertation will be introduced in this chapter. In addition, a model of indigenous culture of teaching and learning is developed and included in this chapter.

Chapter Two is the literature review. There are three topics included in this Chapter, constructivism in math education, NCTM and Chinese MOE Math Standards and Principles, and the indigenous culture of learning and teaching in the USA and China.

Chapter Three is the theoretical models and data analysis. A new cultural model regarding teaching and learning will be established in this chapter. In addition, NCTM 2000 Math Standards and MOE 2001 Math Standards are analyzed through constructivist lenses.

Chapter Four is the methodology of this study. The research design, data collection, and data analysis will be explained in this part.

Chapter Five is the empirical data analysis. The first section will analyze the quantitative data from math teachers' responses in both countries. The second and third sections will analyze qualitative data from China and the USA respectively.

Chapter Six is the conclusions and discussion.

## A Model of Indigenous Culture of Learning and Teaching in China and the USA

In this section, a cultural model of teaching and learning in China and the USA is established for the purpose of analyzing the influence of indigenous culture in comparative studies of teaching and learning in these two countries. In this new model, Confucianism and Taoism emerge as the most influential beliefs and values in terms of teaching and learning in China, in contrast with Behaviorism and Individualism in the USA.

Researchers in comparative education are agreed that the purposes of comparative study can be articulated in two phrases, “to understand” and “to learn from” (Halls, 1990; Holmes, 1971; Lauwerys, 1969; Phillips & Schweisfurth, 2006). The goal of comparatively understanding some aspect of educational practice requires a model of the most influential beliefs and values regarding learning and teaching in the two countries. It is only within the past thirty years that comparative researchers noticed and warned that the national culture plays a fundamental role in the adoption of promising practices from other cultures. Within that period, a few influential cultural models have been developed for purposes of comparative education (e.g., Ho, 1991 cited in Wong, 2004; Hofstede, 1980; Li, 2003, 2004; Walker & Dimmock, 1999). Within my own interest area of mathematics education, many recent studies have specifically factored culture-related issues in comparative analyses (e.g., Li, 2003, 2004; Phuong, Terlouw, & Pilot, 2006; Stigler & Hiebert, 1999; Thomas, 1997). The new model presented here is more specific than the extant cultural models of learning and teaching in China and the U.S.

### Theoretical Framework

Emergence theory is adopted in the definition of culture. Emergentists hold the view of non-reductive materialism. At the level of individual mental functioning, this position maintains that “mental properties are supervenient on the physical brain and yet not reducible to physical properties” (Sawyer, 2001, p. 580). At the societal level, the parallel perspective is that “social

properties are supervenient on individual properties and yet not reducible to those properties” (Sawyer, 2001, p. 580). From this perspective, I perceive indigenous culture as a collection of interconnected social properties realized in the collective activity of a cultural group. It is dynamically stable but subject to gradual change. Beliefs and values about knowledge are an important part of the indigenous culture of teaching and learning that are realized in the activities concerning schooling within a culture.

The following figure illustrates the relationships of cultural elements regarding the indigenous culture of learning and teaching. As a collection of interconnected social properties, culture is manifested as both cultural precepts or principles and cultural practices as shown below. Mutual influence constitutes the dynamic features of culture, as illustrated in Figure 1.2.

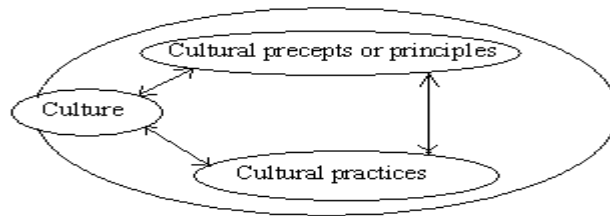


Figure 1.2: Culture and Practice

From the emergentist perspective, cultural precepts and principles are perceived as more stable social properties than cultural practices. Cultural practices are understood as current individual actual practices within a cultural context. This framework implies that one cannot explain individual actual practices completely based on cultural precepts or principles, nor can one fully deduce cultural precepts or principles from examination of individual practice; though the levels of analysis are deeply interdependent. This conception helps resolve conflicting perspectives in comparative research. For example, researchers have argued that Eastern countries and regions, such as China, Hong Kong, Taiwan, Japan, Singapore, Malaysia, and South Korea, are identified as Confucian Heritage Culture (CHC), loosely defined in terms of Confucianism, Taoism, and Buddhism. However, some of the countries and regions also are

influenced by other cultures. Wong (2004) described the CHC disputes in a mathematics education conference in Asia:

A participant from Singapore objected to the classification of Singapore as a CHC region on the grounds that Singapore was a country of multi-cultures... We can also doubt whether Hong Kong, being greatly influenced by Western culture, could be classified as a CHC city...it is not easy to account for the case of Mainland China when traditional culture was once wiped out by Communism. There are 28 provinces and 56 ethnic groups in China. Geographically, it is not easy to identify central China. (p. 511)

The objections and questioning of CHC demonstrates the shortcomings of existing perspectives on indigenous culture. Researchers perceive CHC as a static and structural category. CHC was intended to MATCH with actual practices. From an emergentist perspective, perceiving culture as a collection of interconnected social properties one does not need to object to the existence of other culture influence. CHC can be identified as one of the culture properties, and it can interact with other culture properties in a certain area or country over time. Nor is necessary to match CHC with actual practices—CHC and actual practices can interact together with non-reductive features.

This emergent cultural conception provides an alternative, poststructural, way to establish cultural models that attends to possible interrelations of specific social properties and specific actual practices. In contrast, the extant cultural models tend toward a universalist representation of culture. Hofstede's (1980) model, for instance, contained four dimensions that were perceived as static categories that could be applied to all societies and nations. Although a universal model may be convenient for cultural analysis, it risks disconnection of the dimensions in the static category from actual cultural practices. For example, some Chinese actual practices in teaching and learning reflected Confucian beliefs and values. However, Confucian beliefs and values are outside of Hofstede's four dimensions. My interest is to establish a specific culture model for analyzing teaching and learning in China and the USA that is responsive to actual teaching practices.

## Analysis of the Indigenous Cultural Elements in the New Model

In the new model, Confucianism and Taoism emerge as the most influential beliefs and values in terms of teaching and learning in China, in contrast with Behaviorism and Individualism in the USA. According to the cultural conceptions in Figure 2, we are concerned with the two level interactions, Precepts or Principles and Cultural Practices, when selecting cultural elements in both countries. That is, I first trace back to the histories in both China and the USA in order to find prominent theories, beliefs, and values that have had a long term influence in education. And then I examine if these theories, beliefs, and values are still reflected in current cultural practices in terms of teaching and learning.

### Analysis of Confucian and Taoist Cultural Elements

Notions of Confucianism adopted to this study reference the Confucian precepts in *Analects*, a classical book that best represents Confucius' perspectives on teaching and learning. Taoist ideas adopted in this study reference the Taoist belief in "changing" as incorporated into *yin-yang* principles. Both *Analects* and *yin-yang* principles have heavily influenced Chinese culture with regard to learning and teaching over two thousand years. (A detailed description of this influence will be presented in Chapter Three, Section One.)

Li's (2003) study provides evidence that Confucianism is still a foundational belief regarding learning in current cultural practices. In her study, Li asked Chinese students to generate learning-related words. Many items that Chinese students generated are consistent with Confucius' sayings in *Analects*. For instance, the first item in Li's sorting is "Perfect oneself morally." In *Analects*, moral issues are very important and can be found in many different chapters. In chapter six, Confucius answered his student's question about humaneness, "Now the humane man, wishing himself to be established, sees that others are established, and wishing himself to be successful, sees that others are successful. To be able to take one's own familiar feelings as a guide may definitely be called the method of humaneness" (translated in Dawson,

1993, p. 23). Humaneness is an ideal human quality in Confucianism; this concept is also referred to as perfect virtue, kindness, goodness, human-heartedness, and benevolence. Confucius emphasized that to become a humane man, one must become a model for others first. (In Chapter Three, Section One, other items that Li identified in her study will be analyzed with connection to *Analects*.)

Confucianism is implicitly adopted in the current teaching practices in China. Two outstanding teachers, Qianxiang Zhao and Guimei Dou, introduced similar teaching strategies and beliefs in the late 1990s (Wang, 1999; Zhao, 2000). Most of these strategies and beliefs are consistent with Confucian beliefs and values reflected in *Analects*, although they were not identified as Confucian teachings. These two teachers are recognized as two of the ten outstanding teachers nationwide, the highest honor in teacher education in China. They became models for other teachers. (In Chapter Three, Section One, these two teachers' teaching strategies and beliefs will be analyzed in connection with *Analects*).

The Taoist belief of “changing” is manifested in “teaching with variation”— a current claim of a *Chinese way* for teaching (Gu, Huang, & Marton, 2004). In the actual practices, Chinese teachers also highlight the ideas of “open-ended problem,” “different ways to solve one problem,” “generating different problems from one problem,” etc. Unlike the influence of Confucianism, Taoist principles usually interact with other cultural elements and generate new cultural elements. For instance, “open-ended problem” does not originate in Chinese culture; “generating different problems from one problem” is also found in George Polya's (1957) very influential prescriptions for problem solving. Moreover, as in the West, Polya's ideas regarding problem solving have had a big influence in China's math education since the 1980's. As these ideas shared the common feature “changing,” Chinese scholars and teachers easily adopted them into their theories and practices. In contrast, Behaviorism did not attract Chinese teachers' attention, although it was introduced into China in the 1950s.

## Analysis of Behaviorism and Individualism Cultural Elements

Behaviorism has a long history of impacting learning and teaching in the USA.

Behaviorism holds a Reductionist position (Sawyer, 2002, p. 3). (In Chapter Three, Section One, the major features of behaviorism will be explicated.)

Stigler and Hiebert's (1999) study strongly supported the argument that Behaviorism is still unconsciously influencing current cultural practice. Stigler and Hiebert (1999) analyzed video-taped lessons from the USA, Japan, and Germany as part of the Trends in International Mathematics and Science Study (TIMSS). The authors argued that the typical teaching patterns drawn from the American classroom reflected behaviorist beliefs. (In Chapter Three, Section One, the detailed analysis will be included.)

Researchers have identified Individualism as an important Western belief both in theory and in practice. Oyserman, Coon, and Kimmelmeier (2002) argued that individualism is “a uniquely American characteristic, an integral part of their culture” (p. 4). Even at the beginning, Americans demonstrated the individualist dispositions. As de Tocqueville described, “Such folk owe no man anything and hardly expect anything from anybody. They form the habit of thinking of themselves in isolation and imagine that their whole destiny is in their own hands” (1835/1969, p. 508, cited in Oyserman, Coon, & Kimmelmeier, 2002). Notions of *Individualism versus Collectivism* are often used in comparative studies. For instance, this pair of terms is identified as one of the four dimensions in Hofstede's (1980) cultural model. As well, some items that American students generated in Li's (2003) study reflected an individualist tendency. (A detailed analysis will be included in Chapter Three, Section One.)

## The Functions of the New Cultural Model

This new model provides an alternative way to perceive indigenous culture based on an emergentist perspective (Sawyer, 2001). This conception can solve the conflicting understandings in the extant models (e.g., HCH) as discussed before. The new cultural model

presented in this section serves as a tool to analyze the influence of indigenous culture in comparative studies of teaching and learning in China and the USA.



## CHAPTER TWO: LITERATURE REVIEW

### Constructivism in Math Education

Because there are many complicated interpretations on constructivism in various discourses, in this review I try to focus on these interpretations that have an impact on math education. I first give a brief review on the origins and camps of constructivism, and then move to introduce the most influential work in mathematics education. Following this introduction, I continue narrowing down the topic toward its applications in math education.

#### Origins and Camps of Constructivism

Due to the complicated nature of constructivism in the various discourses, it is very difficult to draw a clear picture on this terminology. Many researchers (Davis & Sumara, 2002; Ernest, 1998; Nelson, 1997; Phillips, 1995) have made efforts to clarify the constructivist metaphor. Most researchers agree that the ideas of constructivism can be traced back to Kant, Vico, Kuhn, and Dewey. These origins are based purely on epistemological claims. That is, all these philosophers provide new perspectives on *knowledge*. For instance, Kant contended that humans' knowledge begins with experience, and "Knowledge is made---constructed---through synthesis, which is performed by applying the categories of pure understanding to what is perceived" (Nelson, 1997). With the concern of how to view knowledge, researchers argued that some philosophers, such as William James, Charles Pierce, Hegel, can be regarded as precursors of constructivism too. Many researchers reached the consensus that the most influential work on constructivism during the last decades were from Piaget, von Glasersfeld, and Vygotsky. In addition, Alcoff and Potter's feminist epistemologies were perceived as constructivist perspectives as well.

To clarify the multiple faces of constructivism, Phillips (1995) posited three dimensions. The first dimension or axis can be simply described as “individual psychology versus public discipline” (p. 7). The first label describes “how the individual learners goes about the construction of knowledge in his or her own cognitive apparatus” while the second label holds that “the individual learner is of little interest, and what is the focus of concern is the construction of human knowledge in general” (p. 7). Based on the features of their works, Phillips (1995) located the possible constructivists’ positions in this axis: Piaget and Vygotsky at the beginning of the line, von Glasersfeld, Kant, and Popper in the middle, and feminist epistemologists at the end of this axis.

Phillips’ (1995) second dimension is “humans the creators versus nature the instructor” (p. 7). This dimension indicates that new knowledge is created by humans, either individually or socially, or imposed from the outside. The beginning of this axis represents the constructivist point of view, while the end of this axis is “minimally constructivist in orientation” (p. 7). The author first analyzed John Locke’s position as a non-constructivist, with von Glasersfeld identified as a constructivist at the individual creation of knowledge, and Barnes, Collins, and Fuller identified as constructivists in the sociopolitical sense. Popper was situated at the middle of this category.

The third dimension Phillips clarified was that “the construction of knowledge is an *active* process, but activity can be described in terms of individual cognition or else in terms of social and political processes (or, of course, in terms of both)” (p. 9). In addition, the activities mentioned here include physical, mental, or both. Piaget, John Dewey and William James were perceived in this axis with the emphasis on individual cognition. Others who insist that activity can be described as social and political processes were Lynn Hankinson Nelson, Barnes, Collins, Martin Hollis, and Imre Lakatos. A majority of these researchers were from feminist research and sociology of knowledge. The author borrowed Latour’s terms to describe this social

constructivism gradation: “radical,” “progressivist,” “conservatives,” “reactionary,” and “golden mean” (p. 10).

Phillips’ (1995) classification focused on epistemological aspects, as shown above. Different angles on constructivism categories can be found in many resources, for instance, Davis and Sumara (2002), Nelson (1997), and Ernest (1994). During the 1980’s, researchers became interested in replacing behaviorism with constructivism, and finally constructivism became prevalent during the 1990’s in mathematics education. As a result, NCTM’s *Principles and Standards for School Mathematics* (2000) has adopted the ideas of constructivism. The next section will discuss how constructivism ideas have been adopted in math education since the 1980’s.

### Constructivism in Math Education

It took about five decades (1950-2000) from the burgeoning up of constructivism to its prevalence in math education. Noddings’ following summary illustrated her intention to frame radical constructivist perspectives in mathematics education:

1. All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
2. There exist cognitive structures that are activated in the processes of construction. These structures account for the construction; that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.
3. Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.
4. Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism.
  - a. Methodological constructivism in research develops methods of study consonant with the assumption of cognitive constructivism.
  - b. Pedagogical constructivism suggests methods of teaching consonant with cognitive constructivism. (1990, p. 10)

In Noddings' review, methodological constructivism indicated assumptions such as “human beings are knowing subjects,” “human behavior is mainly purposive” (p. 7)—these lead to specific methods used in research (e.g., ethnography, clinical interviews, overt thinking). Pedagogical constructivism was implied by cognitive constructivism. In mathematics education, the most influential constructivists are Piaget, von Glasersfeld, Vygotsky. Others (e.g., Steffe, Cobb, Yackel, Richards, Weinberg, Gavelek, Bishop, Ernest) made further contributions in terms of clarifying constructivism, either radical or social, or both, in math education. Furthermore, researchers (e.g., Simon, Steffe, Gravemeijer, Kirshner, Tharp, Gallimore, Confrey) have explored how to make constructivism theory practical. To make the ideas clear, I first introduce radical constructivism and social constructivism in math education, and then I synthesize the debates and important issues. The radical constructivism is based on von Glasersfeld's work, from a Piagetian perspective, while the social constructivism I write about is mainly based on Ernest's and Vygotsky's work. Some other theories (e.g. situated cognition) are not included in this review, although they usually possess the basic features of constructivism epistemology.

### Radical Constructivism in Math Education

The term radical constructivism came to math education discourse in the early 1980's. The representative of this constructivism camp is von Glasersfeld. According to Steffe and Kieren (1994), two von Glasersfeld's articles, published in *Journal of Research of Mathematics Education* in 1980 and 1981, successfully interpreted the radical aspect of Piaget's genetic epistemology. In a later article, *Learning as a Constructive Activity*, von Glasersfeld (1987) explicitly stated both epistemological and psychological aspects of radical constructivism.

Two pairs of words can typically describe the epistemological aspects of radical constructivism: *knowing* versus *knowledge*, *fit* versus *match*. According to von Glasersfeld (1991), “Radical Constructivism is a theory of *knowing*” (p. xv), which denies objective knowledge of the world. Instead of the static status of the traditional theory of knowledge,

knowing is a process through which one can deal with based on his/her experience. This basic assumption leads to a unique way to perceive human being's communication. von Glasersfeld (1991) wrote:

If everyone had a different experiential world, they tend to argue, we could not agree on anything and, above all, we could not communicate, there is not much wrong with that argument, but the fact that we *do* agree on certain things and that we *can* communicate does not prove that what we experience has *objective reality* in itself. If two people or even a whole society of people look through distorting lenses and agree on what they see, this does not make what they see any more *real* (p. xvi).

The above arguments lead to a conclusion that one can construct viable knowledge. von Glasersfeld (1987) claimed that “it is in this context [a teacher models children's concept] that the epistemological principle of *fit*, rather than *match*, is of crucial importance” (p. 13). This epistemological principle of fit also indicates there is no way to access one's reality—the only way we know others is to make hypotheses about the reality. These basic tenets resulted in a new research methodology called “The Constructivist Teaching Experiment” (Steffe, 1991, p. 177) in math education.

In terms of psychological aspects of radical constructivism, von Glasersfeld highlighted motivation in his 1987 article, and the terms from Piagetian tradition such as assimilation, accommodation, perturbation. He claimed it was a misconception that motivation was from the reinforcement such as cookies, money, and social approval. The motivation can be from the inside of the organism's own system in order to achieve a satisfactory organization. For example, when children play puzzles or wooden blocks, the rewards spring from their achievement rather than from the outside. This perspective has been adopted by Simon and Tzur (2004) to explain his activity-effect relationships that served as a mechanism in math concept development (p. 92). Although researchers who held radical constructivist beliefs also adopted Piaget's notions, such as assimilation and accommodation, to explain the process of cognitive construction, they entirely abandoned Piaget's stage theory which was largely criticized by academic community.

The relationships between Piaget and von Glasersfeld constructivism theory can be articulated as follows:

- Both Piaget and von Glasersfeld held the view that knowledge is constructed based on learners' past experience.
- Piaget did not clearly state the notion of constructivism. His main interest was to develop his stage theory. As a psychologist, Piaget took the concept of *adaptation* seriously in his study.
- von Glasersfeld adopted Piaget micro-genetic epistemology to develop his radical constructivism theory. As a philosopher and psychologist, von Glasersfeld took the concept of *self-organization* and the viability principle seriously in his study.
- von Glasersfeld's radical constructivism has integrated in some math education programs developed by Steffe, Cobb, Yackel, Simon, and others. In contrast, the applications of Piaget's theory were not very successful in math education, as Steffe and Kieren (1994) reviewed.

### Social Constructivism in Math Education

Social constructivism, as Ernest pointed out, entered the mathematics education field as a philosophy at the 1980's. In social psychology, Harre, Gergen, Shotter, Coulter, Secord discussed the social construction of the self, personal identity, gender and the like in the 1970's. Researchers (e.g., Weinberg & Gavelek, 1987; Bishop, 1985; Ernest, 1990, 1991a, 1991b, 1994, 1998; Bauersfeld, 1992; Bartolini-Bussi, 1991, Cobb & Yackel, 1996) have conducted studies in mathematics education relying on social constructivism either directly or indirectly. Ernest (1994) clarified the two kinds of social constructivism: social constructivism with a Piagetian theory of mind and social constructivism with a Vygotskian theory of mind. Ernest (1998) offered a deeper analysis of social constructivism as a philosophy of mathematics.

According to Ernest (1998), Wittgenstein's and Lakatos' contributions have been under-recognized. Based on Wittgenstein's language game and Lakatos' logic of mathematics discovery (LMD), Ernest identified "the social construction of subjective and objective knowledge of mathematics" (p. 241). He claimed,

At the center of social constructivism lies an elaborated theory of both individual or subjective knowledge and social or objective knowledge—equally weighted (although in traditional epistemology the latter is prioritized)—and the dialectical relation between them. There is, first of all, a powerful structural analogy between subjective and objective knowledge of mathematics through the role of conversation. For the two types of voice in conversation are those of the knowledge constructor (proponent) and critic, types that figure in the construction and warranting of both personal and public knowledge of mathematics. Second, these types of knowledge are dialectically interrelated and implicated in each other's creation and warranting. (p. 241)

Ernest pointed to a broader sense of math knowledge. He counted Popper's three-world knowledge— physical world, conscious experiences, and contents of books and libraries—as objective knowledge. Therefore, math theories, axioms, problems, conjectures, and proofs are perceived as objective knowledge. In his classification, know-how, propositional knowledge, tacit, and explicit knowledge also belong to math knowledge.

Aside from the concern of the philosophical aspect of social constructivism, we still need to introduce Vygotsky's theory. Vygotsky was influenced by emergence theories that were prominent during the 1930s. Sawyer argued that Vygotsky was a sociological holist, "because he did not attempt to explain social phenomena themselves in terms of how they emerged from individuals and interactions" (Sawyer, 2002, p. 15). This point was reflected in Vygotsky's critique on the element analysis in psychological research. Vygotsky (1934, translated by Hanfmann & Vakar, 1962) argued element analysis can be compared to the chemical analysis of water into hydrogen and oxygen. The whole properties of water cannot be found by analyzing the elements of hydrogen and oxygen.

Instead of element analysis in psychology, Vygotsky developed a new method called *analysis into unites*. He wrote,

By *unite* we mean a product of analysis which, unlike elements, retains all the basic properties of the whole and which cannot be further divided without losing them. Not the chemical composition of water but its molecules and their behavior is the key to the understanding of the properties of water. The true unit of biological analysis is the living cell, possessing the basic properties of the living organism (Vygotsky, 1934, translated by Hanfmann & Vakar, 1962).

Vygotsky's perspective of unite analysis is consistent with the key point held by the social holists that macrosocial phenomena cannot be redefined as individual behavior, a non-reductionist perspective.

Vygotsky's perspective on development can be briefly summarized as "Every function in the cultural development of the child appears on the stage twice, on two planes. First, on the social plane, and then on the psychological; first, between people, and then, inside the child" (Vygotsky, 1987; cited in Vasily V. Davydov, 1995, p. 16). Vygotsky put the social plane as primary, in opposition to Piagetian radical constructivist perspectives on the child development. Unlike the radical constructivist von Glasersfeld, who claimed the construction process was inherently pleasurable for humans, Vygotsky did not believe "that learning related to the zone of proximal development is always enjoyable" (Chaiklin, 2003, p. 43).

Vygotsky's Zone of Proximal Development (ZPD) has been applied in many projects (e.g., Tharp & Gallimore, 1988; Murata & Fuson, 2006, Steele, 2001) in math education, although it is not a central concept of his theory of child development. The following quotation is well documented as the definition of ZPD:

The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. (Vygotsky, 1978, p.86. cited in Chaiklin, 2003, p. 40)

According to Chaiklin (2003), the common conceptions of ZPD can be interpreted as three aspects: generality assumption, assistance assumption, and potential assumption. The first one hypothesizes a student can perform a greater number of tasks in a collaborative situation than he/she can independently. The second one focuses on the positive influences of competent



people in the students' learning situation. The third one inspires the expectation that the child can be accelerated within the potential space. Chaiklin (2003) further summarized that the main features of the analysis of ZPD are the five aspects: a) whole child, b) internal structure, c) development as a qualitative change in the structural functions, d) brought about by the child's actions in the social situation of development, and e) each age period has a leading activity to develop new functions (p. 50).

#### The Debate and Important Issues on both Radical and Social Constructivism

Radical constructivism has caused a hot debate (e.g., McCarty & Schwandt, 2000; Lerman, 1994). Lerman (1994) pointed out two shortcomings of radical constructivism. One is its "troublesome 'social'" (p. 43), and the other is the limitations of von Glasersfeld's description of the semiotic function of math symbols. In terms of radical constructivism's epistemological position, Noddings (1990) argued that "the constructivist assumption should be followed by a break with epistemology. . .the constructivist position is really post-epistemological" (p. 12). Phillips (1995) claimed that von Glasersfeld developed his epistemology in a flawed way and "his individualism and subjectivism in epistemology leads him (or perhaps was accepted because it allows him) to argue that each individual science and mathematics student is responsible for building his or her own set of understandings of these disciplines" (p. 10)—this is seen as problematic for radical constructivism in application to classroom teaching.

Radical constructivists have been actively involved in these debates. von Glasersfeld defended his position from Skepticism and Solipsism. But he totally agreed with Noddings' comments. He wrote, "I agree whole-heartedly with Noddings when she says, at the beginning of her contribution to this volume, that radical constructivism should be 'offered as a post-epistemological perspective. . .constructivism needs to be radical and must explain that one can, indeed, manage without the traditional notion of Truth'" (p. 19). He did not avoid discussing

the “troublesome ‘social’” in radical constructivism—the discussions can be found in his 1991 book *Radical Constructivism in Mathematics Education*.

### Didactic Issues in Mathematics Education

von Glasersfeld’s 1991 book aimed to explore the didactic attitude changes when the authors applied constructivist perspectives in their practice. Among these authors in his book, Cobb, Steffe, and Brink made significant arguments with regards to practice issues. Cobb discussed “troublesome ‘social’” in radical constructivism; Steffe speculated the requirements for constructivism teaching; Brink made comparisons between radical constructivism and Realistic Mathematics education. Cobb admitted that he was influenced by radical constructivism. However, in his teaching experiment, he found that “We will be unable to talk about the specifics of instruction in a theoretically grounded way unless we place analyses of learning within the context of classroom social interactions” (p. 162). Cobb then gave a detailed description of small group interactions in his experiment. These perspectives were further elaborated at Yackel and Cobb’s 1996 paper where they clarified sociomathematical norms in the class. Starting from exploring radical constructivism in classroom practice to adopting both individual and social perspectives in teaching, Cobb’s position is more in social constructivism than in radical constructivism. In this book, Steffe speculated ten principles to adopt constructivism in the research activities or teaching:

1. To learn how to communicate mathematically with students.
2. To learn how to engage students in goal-directed mathematical activity.
3. To learn the mathematics of the students they teach.
4. To learn how to organize possible mathematical environments.
5. To learn the content of variable experiential fields – the mathematical experience of students.
6. To learn the mathematics *for* the students they teach.

7. To learn how to foster reflection and abstraction in the context of goal-directed mathematical activity.
8. To learn how to encourage students to communicate mathematically among themselves.
9. To learn how to foster student motivation and how to sustain learning over a long period of time.
10. To learn how to communicate pedagogically as well as mathematically with other mathematics educators. (Steffe, 1991, p. 191)

Unlike Cobb, who concern with the whole classroom interaction, Steffe's constructivism teaching experiment only involved one student, Maya. So it is unnecessary to discuss social issues in his article. As analyzing his ten principles, I found some principles, such as 4, 7, 8, and 9, still demonstrate the social features. The phrases "foster student motivation," "encourage students to communicate," "to foster reflection and abstraction" indicate the comfortable atmospheres in student-teacher situation. These phrases also echo to Cobb's remark that the trusting relationship is "the most important feature of constructivist teaching" (Cobb, 1991, p. 174). In addition, the principle 2 might be an inspiration for Simon to elaborate his hypothetical learning trajectories. Steffe and Thompson (2000) further elaborated two important notions *mathematics of students* and *students' mathematics*, Students' mathematics is the students' math reality, and mathematics of students is the realities that researchers or teachers assume about the students' reality. Researchers and teachers only can interpret the mathematics of students based on their own experience and information from the literature. Steffe's above principles and notions are consistent with radical constructivism tenets without the "troublesome 'social'."

One of Steffe's statements in von Glasersfeld's book seemed problematic to me. Steffe argued, "The realistic teaching ideas have taken root among the radical constructivists" (Steffe, 1991). Here the realistic teaching refers to Realistic Mathematics Education (RME) from Dutch tradition. The key difference between RME and radical constructivism is the social issue—many

RME researchers identified themselves as adherents of social constructivism. In my analysis, the following aspects are different between RME and radical constructivism:

1. Social interactions versus individual reflections.
2. Didactical phenomenological analysis (Freudenthal, 1983) versus genetic phenomenological analysis.
3. Emphasis on instruction design versus emphasis on theoretical contribution (e.g. Gravemeijer's teaching experiment versus Steffe's constructive teaching experiment).
4. Emphasis on mathematizing (Heuvel-Panhuizen, 2003) and the process from informal to formal versus emphasis on scheme assimilation and accommodation.

A similar point, as Steffe made, was also found in von Glaserfeld's book to compare the similarities between RME and radical constructivism perspectives. It is true that the notion of *realization* in realistic mathematics education is equivalent to "*to construct*" and "*to confront*" (Van den Brink, 1991, p. 199). The basic tenets between theories are far away from each other.

Despite the problems and critiques of radical constructivism in the discourse, it is more coherent as a theory than social constructivism. A big problem for social constructivism is "a lack of consensus about what is meant by the term, and what its underpinning theoretical bases and assumptions are" (Ernest, 1994, p. 63). If we take Ernest's classification, social constructivism with a Piagetian theory of mind and social constructivism with a Vygotskian theory of mind, the following researchers have made their efforts on this domain.

The researchers in the first category:

Richards (1991) and Confrey (1991) took radical constructivism first and then added social interaction.

Murray (1992), Bauersfeld (1992), Cobb, and Yackel (1989, 1996) held the two perspectives should be complementary and interacting.

Ernest (1991a) combined radical constructivism with a fallibilist social theory of mathematics.

The researchers in the second category:

Bartolini-Bussi (1991, 1994), Murata and Fuson (2006), Steele (2001), Weinberg and Gavelek (1987) conducted research based on Vygotsky's theory.

Ernest (1993a, 1993d, & 1994) revised his 1991a version of constructivism—there is no underlying metaphor for the isolated individual mind.

Researchers in the first category acknowledged that knowledge is constructed both individually and socially, while researchers in the second category contended that knowledge is constructed socially. In contrast, radical constructivists only acknowledged that knowledge is constructed individually.

In short, the key questions in the debate are “whether mind is located in the head or in the individual-in-social-action, and whether development is cognitive self-organization or enculturation into established practices” (Cobb, 1994, p. 13).

### Constructivist Teaching in Math Education

Although constructivism has provided mathematics educators with useful ways to understand learning and learners, the task of reconstructing mathematics pedagogy on the basis of a constructivist view of learning is a considerable challenge. One that the mathematics education community has only begun to tackle. (Simon, 1995, p. 114)

During the last two decades, the insightful works on exploring constructivism teaching are Simon's Hypothetical Learning Trajectory (HLT) and its elaboration, Cobb and Yackel's sociomathematical norms, Kirshner's Crossdisciplinary Framework, and Murata and Fuson's ZPD Mathematical Proficiency Model. Simon's HLT has led to a discussion in terms of learning trajectories in various theoretical foundations (Clements & Sarama, 2004). Cobb and Yackel's sociomathematical norms made them toward a social constructivism orientation. Kirshner's Crossdisciplinary Framework provided a new angle to reconceptualize constructive metaphor.

Murata and Fuson's ZPD mathematical Proficiency Model was an extension of Tharp and Gallimore (1988) framework based on Vygotsky's theory.

### Kirshner's Crossdisciplinary Framework

Kirshner's Crossdisciplinary Framework (2002, & 2008) is mainly concerned with three important notions, skills, concepts, and dispositions as explored in various psychological traditions. Three learning metaphors, *Habituation*, *Construction*, and *Enculturation*, have been separately drawn out from our usual integrative discourse. Habituation learning concerned with skill acquisition; Construction learning concerned with concept development; and Enculturation learning concerned with inculcating dispositions. Constructivist theory in math education discourse has been adopted for the Construction metaphor in Crossdisciplinary Framework, particularly, Piaget's microgenesis theory and radical constructivist perspective.

In terms of constructivist pedagogy, Kirshner developed two forms, student-centered and teacher-centered pedagogies. For student-centered constructivist teaching, Kirshner contended the following aspects:

The teacher needs to have a model (always tentative) of the student's current conceptual structures, including the limitations of those structures relative to a mature understanding of the particular content to be taught . . . then the teacher helps mediate the student's engagement with the task by (1) monitoring the student's uptake of the task, making minor adjustments to it, as needed, (2) assessing the effectiveness of the task in stimulating development, as intended; this may involve rethinking and revising the model of the student's understanding, and/or the task environment, (3) responding to the students as they engage with the task to help them experience the discrepancies more fully, and (4) encouraging the student through the frustration that arises when conceptual obstacles are encountered. (Kirshner, 2008, p. 14)

For student- centered pedagogy, curriculum focuses on math concept rather than math topics. It is claimed that teachers/researchers should have a mature model of the concept, and chart out a hypothetical learning trajectory for the student. The four mediating strategies in student-centered pedagogy demonstrate the ways of approaching *mathematics students*, which are compatible with the principles in Simon's Hypothetical Learning Trajectories. These steps

also can be perceived as an elaboration of Zone of Potential Construction (ZPC) (Steffe, 1995). Steffe's ZPC mainly contains two aspects: 1) the teacher's hypotheses of the students' schemes of action and operation at the forefront, and 2) the actions of students elicited by the situation at the foreground (Steffe, & D'Ambrosio, 1995, p. 154).

For teacher-centered constructivist pedagogy, Kirshner argued that lecture is an appropriate format for teaching "whenever the student's metacognitive sophistication is sufficient to accommodate the gap between current and mature forms of the concept" (Kirshner, 2008).

One of the advantages of the constructivist pedagogy in crossdisciplinary framework is that the teacher can grasp the teaching goal with convincing learning principles—this is usually a deficit in integrative discourse—there is no clear version on what accounts for good teaching. For example, the teaching goals become blurred when the teaching formats (e.g., individual, small group, whole class) change in integrative discourse. Radical constructivists also cannot well explain how radical constructivist principles benefit small group or the whole class learning. The common solution for this problem was that, if learning occurred individually, it can be explained adequately by radical constructivist principles; if learning occurred in small group or in the whole class interactions, it was usually explained by social constructivist tenets (e.g., Simon, 1995; Cobb, 1991).

In Crossdisciplinary framework, the radical constructivist stance with Piaget's micro-genesis theories of conceptual development can adequately explain learning no matter how many students are involved. This pedagogical elaboration helps teachers focus on the learning goal and a model of the concept rather than the teaching format. The core for constructivist teaching is to facilitate students' conceptual understanding.

Lecture is usually connected with teacher-centered pedagogy in a traditional sense. And over one hundred years, educational reformers have continuously made their efforts on switching

teacher-centered pedagogy to student-centered pedagogy in the USA (Ducharme, & Kluender, 1989). Therefore, lecture is devalued in the integrate discourse, though it continues to be widely practiced. If a teacher adopts lecture for teacher-centered constructivist pedagogy, the role of the lecture is to explain the mature model of the concepts. The requirement for effective use of lecture for constructivist teaching is that students are metacognitively sophisticated—the students are able “to accommodate the gap between current and mature forms of the concept” (Kirshner, 2008, p. 18).

For lower level concepts, the students can benefit from lectures even they are not sophisticated metacognitively. A similar pedagogical method, demonstration, also is included in the crossdisciplinary framework. Demonstration serves the goal of skill development. Demonstration can make salient certain elements in the task domain to facilitate the sub-cognitive correlations that constitute the skill.

#### Murata and Fuson’s ZPD Mathematical Proficiency Model

Murata and Fuson’s (2006) ZPD Mathematical Proficiency Model provides a typical example of the social constructivism application in math classrooms, with a concern of skill acquisition. The model identified two kinds of learning activities: instructional conversations and practice. The former aimed to assist students’ understanding while the latter was to develop fluency. In their article, Murata and Fuson defined teaching as “Teaching occurs when *responsive* assistance is offered by *more capable others* at points at which performance *and understanding* require assistance” (p. 423). This definition is an extension of Tharp and Gallimore’s perspective where assistance is specified by four stages:

Stage I is assistance provided by more capable others, Stage II is assistance provided by the self (as the means of assistance of others are internalized into speech-for-self), Stage III is internalization-automatization-fossilization, and Stage IV is de-automatization with recursion through the stages as performance that was once mastered slips away over time. (p. 424)



Murata and Fuson further specified these stages with some revisions and used it in a Japanese first grade class. They discussed in details that “the teacher decreased assistance over time but increased it for transitions to new problem types and for students who needed it” (p. 421).

#### Simon’s Hypothetical Learning Trajectory and Its Elaboration

Simon published an article dealing with Hypothetical Learning Trajectory (HLT) in 1995. The purpose of this work was to provide “models of teaching based on constructivism” (Simon, 1995, p.114). Although constructivist perspectives became an important issue (e.g., it influenced NCTM 1989’s and 1991’s standards), “the task of reconstructing mathematics pedagogy on the basis of a constructivist view of learning is a considerable challenge” (p. 114). In 2004, he elaborated the HLT by providing a mechanism for thinking about the learning process. The detailed descriptions of Simon’s perspectives are as follows.

As Simon (1995) pointed out, the HLT “is made up of three components: the learning goal that defines the direction, the learning activates, and the hypothetical learning process—a prediction of how the students’ thinking and understanding will evolve in the context of the learning activities” (p. 136).

The word “trajectory” referred to a path. To give an intuitive understanding about the HLT, Simon presented a sailing example. If a person decides to sail around the world in order to visit his unknown places, he would first make a plan for the whole or part of the trip based on his knowledge. And then he starts his sailing. As he faces unpredictable conditions, he needs to constantly adjust the plan. He may add destinations that do not include in his original plan. Simon described, “The path that you travel is your ‘trajectory.’ The path that you anticipate at any point in time is your ‘hypothetical trajectory’” (p. 137).

The HLT was initially drawn out from Simon’s Construction of Elementary Mathematics Project. Before he delivered his lessons to his pre-service teachers from constructivist

perspectives, he first established his initial goal, namely, “students would understand the relationship of multiplying length by width to the evaluation of the area of a rectangle” (p. 132). For Simon, understanding these relationships needed to connect “an understanding of multiplication-as-repeated-addition with the notion of identical rows of units of area and understanding the relationship between linear units and area units” (p. 134). From his past teaching experience, he knew most students would be able to use the formula  $A=l \times w$ , but without understanding the concept of area. Concrete experience with rectangles can help students gain their conceptual understanding. The general method for understanding area is that a small square unit or rectangular unit can be fit on the big rectangle to be measured. Giving only one small tile will make students seek new ways to measure which go beyond counting all the tiles. He also guessed what students’ understanding would be based on his past teaching experience and his knowledge of the literature. These initial ideas led to his problem 1:

Rectangle problem 1. Determine how many rectangles, of the size and shape of the rectangle that you were given, could fit on the top surface of your table. Rectangles cannot be overlapped, cannot be cut, nor can they overlap the edges of the table. Be prepared to describe to the class how you solved this problem. (p.123)

As this problem was presented to the whole class, there were a lot conceptual flaws coming from the students’ responses. For instance, many students were puzzled by Simon’s question “why they had multiplied these numbers?” or puzzled if maintaining orientation of the small rectangle was necessary (figure 1a and 1b, p. 123). It was very important for the teacher to have a deeper understanding of the concept here. Otherwise, he would be unable to analyze his students’ conceptual mistakes. From the HLT perspective, at the above stage, the teacher has entered the second cycling of his/her math teaching. The first one occurred before presenting his/her problem 1, which was a rough outline on his/her teaching goal, students learning activities, and possible learning trajectory that might occur in the class.

The difference of this work from traditional teaching was that Simon did not have a large set of predetermined problems that must be presented in the class. He adopted an inductive attitude to determine where he should go after the students exposed their limits of understanding concepts (e.g. counting, linear unit, square unit, etc.). The second problem Simon presented the class was from students' conceptual mistakes that were not predicted before his class: "Problem 2: Bill said, 'if the table is 13 rectangles long and 9 rectangles wide, and if I count 1,2,3, ..., 9 and then I multiply,  $13 \times 9$ , then I have counted the corner rectangle twice,' respond to Bill's comment" (p. 125). And based on the discussions in the class, Simon found his students were unclear about conceptual understanding of area. He felt it was necessary to pose the third problem: How can you find the area of a figure that is randomly shaped by a closed string (p. 128)? The prerequisite for selecting the problems (and creating situations for students' learning) is teachers' conceptual goals for their students. These goals will be constantly modified as the situation changed rather than changed during planning between classes. To my understanding, the goal is a crucial element among the three in the HLT. The second element, teacher's plan for learning activities, is determined by both the first the third elements. Simon usually performed the second element by asking questions or posing problems. He heavily emphasized teacher's role in the learning process.

In short, the HLT has the following features:

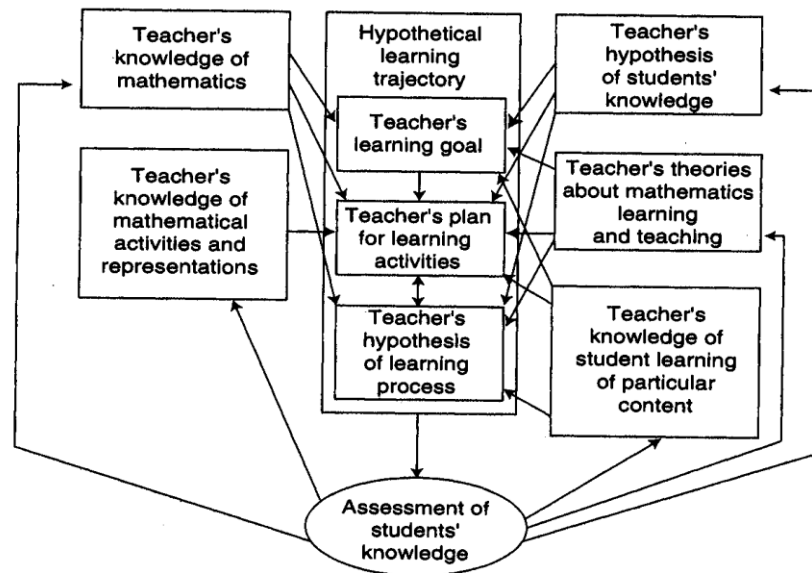
(1) A living model,

It heavily depends on the conceptual goals and students' responses in the learning situation, as Simon (1995) pointed out, "the continually changing knowledge of the teacher creates continual change in the teacher's hypothetical learning trajectory" (p. 141).

(2) Taking students thinking seriously,

Teachers should predict students' mathematics and revise their original predictions based on students' reactions in the class situation.

- (3) Teacher's knowledge grew when teaching in the different situations,  
Teachers are as learners.
- (4) Planning for instruction necessary,
- (5) The need of the experienced teachers to apply this model in the classroom teaching.



*Figure 5. Mathematics teaching cycle. (The domains of teacher knowledge also inform “assessment of students' knowledge” directly. However, because this was not the emphasis of the model, and in order to simplify the diagram, those arrows are not included.)*

Figure 2.1: Mathematics Teaching Cycle

Figure 2.1 also demonstrates the relationships of the three elements of the HLT and teacher's knowledge of mathematics, teacher's hypothesis of students' knowledge, teacher's theories about mathematics learning and teaching, teacher's knowledge of student learning of particular content, and teacher's knowledge of mathematical activities and representations.

Since “the description of the HLT stopped short of providing a framework for thinking about the learning process and the design or selection of mathematics tasks” (Simon & Tzur, 2004, p. 92), Simon and Tzur demonstrated a framework (reflection on activity-effect relationships). They used equivalent fractions as an example to show how this framework

functions in the second article. The second article was entitled as *Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory*. In this article, Simon and Tzur's activity-effect relationships was perceived as an elaboration of reflective abstraction. They wrote,

First, following von Glasersfeld (1995), we assumed that learners have inborn abilities and tendencies (e.g. creating records of experience, sorting and comparing records, and identifying patterns in those records). Second, we assumed that, based on assimilatory conceptions available at the outset, learners have the ability to set a goal, select an activity that was learned previously, and monitor progress toward the goal. (p. 95)

One needs to notice that the learner's goal might be different from teacher's instruction goal. In a game activity, for instance, the instructional goal probably is set for certain conceptual understanding. But the students' goal may be to win the game. Simon told us the learners acquired advanced concepts were within the learners' goal-directed learning process. In the second part of this article, Simon provided an example to explain how this perspective was served as a framework for thinking about the learning process and the role of the task in the process. To my understanding, the elaboration of the HLT indicated that the goal, the activity, and the task were coherently connected to this framework, which can be seen by Simon's example involving equivalent fractions. The following sequences were given in this lesson:

1. Draw a rectangle with  $\frac{1}{2}$  shaded. Draw lines on the rectangle so that it is divided into sixths. Determine how many sixths are in  $\frac{1}{2}$ .
2. Draw a rectangle with  $\frac{2}{3}$  shaded. Draw lines on the rectangle so that it is divided into twelfths. Determine  $\frac{2}{3} = ?/12$ .
3. Draw diagrams to determine the following:
  - a.  $\frac{3}{4} = ?/8$
  - b.  $\frac{4}{5} = ?/15$
  - c.  $\frac{3}{4} = ?/20$

4. Drawing diagrams to solve equivalent fractions problems is not much fun when the numbers get large. For the following do not draw a diagram. Rather describe what would happen at each step if you were to draw a diagram. Use that thinking to answer the following:

a.  $\frac{5}{9} = \frac{?}{90}$

b.  $\frac{7}{9} = \frac{?}{72}$

5. Without drawing a diagram, think in terms of cutting up a rectangle. Use a calculator to calculate the following. Write down each step that you do and the result you get.

Justify each step in terms of how it is related to cutting up a rectangle.

a.  $\frac{16}{49} = \frac{?}{147}$

b.  $\frac{13}{36} = \frac{?}{324}$ .

There was a clear intention for this design. The first three questions were anticipated to lead students to reflect on activity-effect relations, that was, “subdividing the parts into  $x$  subdivisions results in  $x$  subdivisions in each shaded part or  $x$  times the original number” (p. 98). It was not necessary for the students to deal with this relation reflectively (or consciously) at this period. The fourth problem was to enforce students to increase their awareness of the activity-effect relation. Even they might perform well on the previous problems, they might not notice the quantitative relation between the old and the new numerators (e.g. a reflective abstraction from a concrete diagram to quantitative relations). For the last problem, the students were not allowed to draw a diagram. They used calculators to figure out the quantitative relations. In this activity-effect oriented task design, the tasks were selected by gradually increasing the awareness of students’ reflective abstraction rather than a random selection. As Simon and Tzur summarized in this article, “with this elaboration of the HLT, the selection of tasks is not left to intuition or trial and error. Rather, the mechanism offers a framework for thinking about how the task can promote the learning process” (p. 101).

The effects Simon talked about were represented by quantitative relation or patterns that can attract students and contribute to the intended learning. For example, he explained an effect in the first activity was that “a division of the new denominator by the old denominator” (p. 100). The conceptual goal for the lesson design was broken down for seeking specific effects at different abstractive levels of the task. Moreover, the effects were always embedded in the certain activities.

Inspired by Simon’s work, many researchers have conducted their research on mathematics learning and teaching based on the constructs of learning trajectories. A full discussion in terms of learning trajectories have been collected in *Mathematical Thinking and Learning*, 6(2), 2004, where I found that the constructs of learning trajectories has different theoretical foundations. For example, Gravemeijer interpreted hypothetical learning trajectories from RME perspective. Steffe discussed the possibility to construct learning trajectories of children from radical constructivist perspective. Clements and his colleagues developed a specific HLT by figuring out seven levels in the developmental progression for the composition of shapes. Battista mainly introduced a new assessment project, cognition-based assessment (CBA), which was a HLT-like conceptual framework. Lesh and Yoon visualized learning trajectories as a genetic inheritance tree from social constructivist perspective and Darwinian evolution theory. Among these hypothetical learning trajectories, I briefly introduce Steffe’s and Gravemeijer’s work in the following paragraphs.

#### Steffe’s Learning Trajectories of Children

In his article, Steffe provided his six teaching episodes to report and analyze how two children, Jason and Laura, developed their fraction schemes when they engaged in the activities designed for constructivist teaching experiment.

Although Steffe used Simon’s term HLT, they had quite different emphasis when they explained HLT in their own experiments. For Simon, a teacher using HLT in classroom must

establish the conceptual goal and students' possible learning trajectories as well. In contrast, Steffe seemed to weaken teacher's role by emphasizing "learning trajectories of children" (p. 130). He doubted "who is the teacher, and whose responsibility is it to construct learning trajectories?... rather than consider my own knowledge of how children learn mathematics as 'good enough,' and thereby consider the construction of learning trajectories as the responsibility of practicing teachers" (p. 130). To strictly meet radical constructivism principle, Steffe used *trajectories* (p. 130) rather than *trajectory* indicating that each child has his/her own unique construction on the reality. In this article, Steffe only dealt with two children and gave the detailed descriptions of their learning trajectories. In Simon's 1995 article, Simon dealt with more than two children, with which Steffe was struggling (e.g., in a critique (Steffe, 1995, p.151), Steffe argued that Simon's HLT was very similar to Vygotskian approach).

#### Gravemeijer's Hypothetical Learning Trajectories

For Gravemeijer, hypothetical learning trajectories should make use for "the planning of instructional activities in a given classroom on a day-to-day basis" (p. 107). The rationale of Gravemeijer's hypothetical learning trajectories was *local instruction theories* that was based on RME and developed by the new instructional design research (e.g., Gravemeijer 1994, 1998, Gravemeijer, & Cobb, 2001). Gravemeijer took the similar statement as Simon's HLT. Both Simon and Gravemeijer emphasized the teachers or researchers should first set up the learning goals. For Simon, the next step should envision the students' conceptual understandings of the certain topics. In contrast, Gravemeijer provided a detailed description regarding activities and tools for the students' learning by using RME as a framework. In his article, Gravemeijer did not provide detailed information to show how the conjectured learning trajectory (in his term) changed, though he posted the term "accumulative cyclic process" (p. 110) that resembled Simon's "mathematics teaching cycle" (p. 110). In short, RME researchers took more serious consideration of the original design of learning trajectory than constructivists did.



## The USA NCTM and Chinese MOE Math Standards and Principles

In this section, I will provide a literature review on both NCTM 2000 principles and standards for school mathematics (hereafter called the NCTM 2000 standards) and the Chinese Ministry of Education 2001 Mathematics Curriculum Standards for 1-9 Compulsive Education (hereafter called the MOE 2001 standards). A brief introduction on the math standards development in China and the USA is also included in this review. At the end of this review, the similarities and differences between the two math standards are compared and discussed.

### Overview of the *Principles and Standards for School Mathematics*

#### A Brief Review on Math Reform in the USA

Featured as a de-centered education system, the U.S. math education reforms have been debated hotly for nearly one-hundred years. From the early 20<sup>th</sup> century until the 1950s, American math education was dominated by the conflict between progressive education beliefs and pedagogical methods aligned with behaviorism. However, perspectives from progressivism and behaviorism with regard to teaching and learning were not the monolithic voice in math education before 1950. In 1923, NCTM disseminated the report, *The Reorganization of Mathematics for Secondary Education*, which was written by mathematicians and prominent teachers. This report included a survey of math curricula, the training of math teachers in other countries, issues of math learning related to the psychology, and the intrinsic value and application of mathematics. It was also claimed that algebra was very important for every student, a view that was opposite to the progressivists'.

From 1957 to 1970, the New Math Movement prevailed in the U.S. Unlike Progressive Math, curriculum in the New Math Movement emphasized coherent logical explanations. Calculus courses were introduced at the high school level. A fatal weakness of the New Math curriculum is that the math content was extremely formal (Klein, 2003). Set theory and exotic topics were introduced without considering students' previous experience. Basic knowledge and

math applications were ignored. In the early 1970s, New Math Movement was replaced by “back to the basics.” In the 1980s, NCTM published a remarkable report, *An Agenda for Action*. Problem solving was a heart of this report. From 1989 to 2000, NCTM published serious standards documents, and these documents heavily influenced math educational reform. In 1989, NCTM published *Curriculum and Evaluation Standards for School Mathematics*. In 2000, NCTM published *Principles and Standards for School Mathematics*. To some extent, NCTM 1989 standards was an elaboration of its 1980’s *An Agenda for Action*, while NCTM 2000 standards was an update of its 1989’s standards. Two complementary materials were published during this period. One was *Professional Standards for Teaching Mathematics* (1991), and the other was *Assessment Standards for School Mathematics* (1995). The philosophical foundation of NCTM’s math standards (1989, 2000) was consistent with constructivist beliefs and a part of progressive education beliefs.

#### A Review on the *Principles and Standards for School Mathematics* (NCTM 2000)

In 2000, NCTM published *Principles and Standards for School Mathematics*, an update of its *Curriculum and Evaluation Standards for School Mathematics* (1989). The core belief presented in this document is that “all students should learn important mathematical concepts and processes with understanding” (p. ix). Six principles and ten standards for school mathematics are clarified to achieve this goal. The six principles comprise equity, curriculum, teaching, learning, assessment, and technology. The ten standards include five content standards and five process standards. The five contents are number and operations, algebra, geometry, measurement, and data analysis and probability. The content standards explicitly address the content that students should learn. The process standards incorporate problem solving, reasoning and proof, communication, connections, and representation. The process standards recommend the “ways of acquiring and using content knowledge” (p. 29). In this review, I only focus on

introducing six principles and five process standards, since these principles and standards provide the core viewpoint on constructivist teaching and learning.

NCTM claims that the six principles reflect the underlying assumptions, values, and evidence that are very important for a high-quality mathematics education. A brief summary of the six principles is as follows:

NCTM's commitment to mathematics for all is reaffirmed in the Equity Principle. In the Curriculum Principle, a focused curriculum is shown to be an important aspect of what is needed to improve school mathematics. The Teaching Principle makes the case that students must have opportunities to learn important mathematics under the guidance of competent and committed teachers. The view of learning that is the basis for the document is taken up in the Learning Principle. The important roles of assessment and technology in school mathematics programs are discussed in the Assessment and Technology Principles. (NCTM, 2000, p. 7)

### The Equity Principle

The Equity Principle means that all students must have opportunities to learn mathematics. Teachers must have high expectations and strong support for all students. Schools need to establish strong instructional programs to support all students' learning. To reach the goal of educational equity does not mean to provide identical instruction to all students. Rather, NCTM lists a variety of instructional needs for different types of students (e.g., gifted students, students with disabilities, students whose native languages are not English). Further requirements to achieve this goal include the professional development of teachers and resources in schools and classrooms.

### The Curriculum Principle

The Curriculum Principle comprises three major ideas about mathematics curriculum. The first is that a mathematics curriculum should be coherent. NCTM envision that "A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills" (p. 15). Specifically, teachers are recommended to seek coherence

both within a lesson plan and between lesson plans. In addition, teachers are also advised to adjust and take advantage of opportunities to move lessons in unanticipated directions.

The second major idea in the Curriculum Principle is that a mathematics curriculum should focus on important mathematics. Important mathematics refers to those topics that can help develop other mathematical ideas, link different areas of mathematics, and deepen students' appreciation of mathematics. Mathematics concepts and reasoning processes are also considered important.

The claim that a mathematics curriculum should be well articulated across the grades is perceived as the third idea in the Curriculum Principle. This idea has been demonstrated in NCTM's content standards that contain five domains across the grade bands. Although the same five domains are in each grade band, the requirements are different. For instance, K-2<sup>nd</sup> students are required to explore similarities and differences among two-dimensional shapes. Students in sixth to eighth grades are to learn properties of particular quadrilaterals.

### The Teaching Principle

The Teaching Principle focuses on teachers' understanding of what students know and need to learn. NCTM points out three aspects of effective teaching: 1) Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies; 2) Effective teaching requires a challenging and supportive classroom learning environment; and 3) Effective teaching requires continually seeking improvement (p. 19). In terms of teachers' knowledge, NCTM lists the following:

Knowledge about the whole domain; deep, flexible knowledge about curriculum goals and about the important ideas that are central to their grade level; knowledge about the challenges students are likely to encounter in learning these ideas. Knowledge about how the ideas can be represented to teach them effectively; and knowledge about how students' understanding can be assessed. (p. 17)

NCTM emphasizes that effective teaching requires teachers' understanding of students' prior knowledge. Teachers are able to ask questions and plan lessons that reveal students' prior knowledge.

Different strategies and teaching styles are encouraged for helping students' learn. In other words, "There is no one 'right way' to teach" (p. 18). To build mathematics understanding for their students, teachers "must balance purposeful, planned classroom lessons with the ongoing decision making that inevitably occurs as teachers and students encounter unanticipated discoveries or difficulties that lead them into uncharted territory" (p. 18).

To create a challenging and supportive learning environment, teachers need to convey an equity belief for all students. Teachers may ask themselves, "Are students' discussion and collaboration encouraged? Are students expected to justify their thinking?" (p. 18). Aside from a supportive learning environment, effective teaching requires worthwhile mathematical tasks. Such tasks must have challenges and intrigue students to work hard. NCTM highlights different ways to approach a worthwhile math task. For instance, students are encouraged to use an arithmetic counting, draw a geometric diagram, enumerate possibilities, or use algebraic equations. Selecting a worthwhile task is not enough to conduct effective teaching. Indeed, teachers still face the following challenges: 1) how to organize and orchestrate the work of the students, 2) what questions to ask to challenge those with varied levels of expertise, and 3) how to support students without taking over the process of thinking for them and thus eliminating the challenge (p. 19).

To improve teachers' mathematics instruction, teachers are requested to reflect and analyze their work alone or with teams that contain experienced and respected colleague, new teachers, or a community of teachers. NCTM mentions that "Collaborating with colleagues regularly to observe, analyze, and discuss teaching and students' thinking or to do 'lesson study' is a powerful, yet neglected, form of professional development in American schools" (p. 19).

### The Learning Principle

The Learning Principle mainly deals with the issue of learning mathematics with understanding. NCTM adopts Bransford, Brown, and Cocking's 1999 perspectives to state conceptual understanding. That is, conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility. The alliance of factual knowledge, procedural proficiency, and conceptual understanding makes all three components usable in powerful ways (p. 20). Memorizing fact and procedures without understanding are disregarded by NCTM, since such learning makes students not sure when and how to use what they know. In contrast, it is easier to remember and to apply when students' learning is grounded in meaningful ways.

The Learning Principle conveys the message that students can learn mathematics with understanding. Children learn math ideas in everyday life in an informal way. School math needs to build on their experience. Classroom interactions are crucial for students to understand concepts. These interactions include proposing math ideas and conjectures, learning to evaluate their own thinking and others, and developing math reasoning skills. Moreover, problem solving, reasoning, and argumentation can help students develop both procedural fluency and conceptual understanding in such interaction settings.

The Learning Principle also highlights students' dispositions including: 1) confidence in their ability to tackle difficult problems, 2) eagerness to figure things out on their own, 3) flexibility in exploring mathematical ideas, 4) trying alternative solution paths, 5) willingness to persevere, and 6) attempting to learn from their mistakes (p. 21).

### The Assessment Principle

NCTM claims that formal assessments are not the only way to use. Teachers should use assessments for their students, rather than to their students. Assessment should enhance students' learning as well as facilitate teachers' decision making in their classroom instructions. Multiple

techniques of assessment are encouraged, including open-ended questions, constructed-response tasks, selected-response items, performance tasks, observations, conversations, journals, and portfolios (p. 23). Teachers are advised to integrate assessment into instruction—this is described in a constructivist way:

In addition to formal assessments, such as tests and quizzes, teachers should be continually gathering information about their students' progress through informal means, such as asking questions during the course of a lesson, conducting interviews with individual students, and giving writing prompts. . . . The instructional decisions made by teachers—such as how and when to review prerequisite material, how to revisit a difficult concept, or how to adapt tasks for students who are struggling or for those who need enrichment—are based on inferences about what students know and what they need to learn. (p. 23)

### The Technology Principle

The Technology Principle mainly introduces how technology enhances and supports effective mathematics teaching and learning. Technology also influences what mathematics is taught in schools.

Aside from the above six principles, the five process standards in NCTM 2000 standards are very important for understanding its beliefs regarding teaching and learning. The five process standards include problem solving, reasoning and proof, communication, connection, and representation standards.

### Problem Solving

NCTM defines problem solving as “engaging in a task for which the solution method is not known in advance” (p. 52). Problem solving is not only perceived as a way to develop mathematical understanding, but also as a way to do mathematics. NCTM sets up four standards for K-12 students' problem solving:

- Build new mathematical knowledge through problem solving;
- Solve problems that arise in mathematics and in other contexts;
- Apply and adapt a variety of appropriate strategies to solve problems;

- Monitor and reflect on the process of mathematical problem solving.

To meet the first problem solving standard, teachers are required to choose worthwhile problems and to examine if these problems help to further the mathematical goals. In addition, NCTM contends that “problem solving can and should be used to help students develop fluency with specific skills” (p. 52). Followed by this statement, a typical example is provided to demonstrate a problem solving situation can help students acquire addition skills.

To meet the second problem solving standard, teachers need to help students build dispositions, naturally analyzing situations in mathematical terms and posing problems based on the situations. The ways fostering students’ dispositions include asking students questions in order to help them find math in their world and encouraging students to persist with problems. Teachers should create and maintain a supportive environment to encourage students to explore, take risks, share failures and successes, and question one another.

The strategies recommended by NCTM in the third problem solving standard are as follows: 1) using diagrams, 2) looking for patterns, 3) listing all possibilities, 4) trying special values or cases, 5) working backward, 6) guessing and checking, 7) creating an equivalent problem, and 8) creating a simpler problem (p. 54). Following this list, NCTM provides a vision for teaching:

In the lower grades, teachers can help children express, categorize, and compare their strategies. Opportunities to use strategies must be embedded naturally in the curriculum across the content areas. By the time students reach the middle grades, they should be skilled at recognizing when various strategies are appropriate to use and should be capable of deciding when and how to use them. By high school, students should have access to a wide range of strategies, be able to decide which one to use, and be able to adapt and invent strategies...strategies are learned over time, are applied in particular contexts, and become more refined, elaborate, and flexible as they are used in increasingly complex problem situations. (p. 54)

The fourth problem solving standard calls for meta-cognition—the reflection of what they are doing to self-assess their strategies. To develop such reflective habits, NCTM



recommends that teachers should ask questions such as “Before we go on, are we sure we understand this?” “What are our options?” “Do we have a plan?” “Are we making progress or should we reconsider what we are doing?” “Why do we think this is true?” Such habits should be developed in the lowest grades.

### Reasoning and Proof

Reasoning and Proof standards contain four aspects:

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof. (p. 56)

The first aspect states the importance of reasoning and proof in mathematics. The second aspect describes how teachers help students make conjectures in different grade levels. For instance, in lower grade levels, teachers can ask questions such as “What do you think will happen next?” or “What is the pattern?” or “Is this true always?” (p. 57) to help students make conjectures. Teachers also are able to help students rethink conjectures from one context to another context. The third and fourth aspects provide the development levels of mathematical arguments and proofs. In the lower grades, children can use specific cases to justify general claims. They may use trial-and-error strategies or unsystematic trying in their reasoning and proofs. “By the upper elementary grades, justifications should be more general and can draw on other mathematical results” (p. 58). High school students could make complex chains of reasoning. In short, at all levels, students can use patterns and specific cases to think inductively. As their grades increase, they are expected to use deductive reasoning effectively.

### Communication

Communication standards contain four aspects:

- Organize and consolidate their mathematical thinking through communication.

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely. (p. 60)

The first aspect claims communication benefits students in the following: learning of new math concepts, identifying misconceptions, sharing responsibility with teachers, gaining proficiency, and consolidating students' thinking by writing. The second aspect suggests that teachers should build a community "in which students will feel free to express their ideas" (p. 61). There are different requirements for students in different grades in terms of thinking coherently and clearly. Students in the lower grades need more help from their teachers to share ideas than students in other grades. Students in 3-5 grades can participate in whole class discussions. Teachers should create communication-rich environments for middle school students, although they do not like to stand out for group discussions. In addition, written communication should be gradually emphasized from informal to formal as the grades increase. In terms of communicating math ideas and reasoning, NCTM provides a clear statement from the lower grades to the high school.

As students mature, their communication should reflect an increasing array of ways to justify their procedures and results. In the lower grades, providing empirical evidence or a few examples may be enough. Later, short deductive chains of reasoning based on previously accepted facts should become expected. In the middle grades and high school, explanations should become more mathematically rigorous and students should increasingly state in their supporting arguments the mathematical properties they used. (p. 62)

The third aspect in communication standards indicates that students can learn from others in the process of working on problems through communication. Indeed, being able to evaluate others' ideas and methods is perceived as an important ability for students.

The fourth aspect in communication standards suggests that teachers should help lower grades students discriminate the same words with different meanings as they are used in

mathematical expression. Teachers also should avoid “a premature rush to impose formal mathematical language” (p. 63).

### Connections

Connections in the process standards include three aspects:

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics. (p. 64)

The first aspect states that emphasizing mathematical connections can change the view of mathematics as a set of disconnected, isolated rules, concepts, and skills. This aspect is perceived as developing students’ dispositions by NCTM. For instance, NCTM suggests, “Throughout the pre-K-12 span, students should routinely ask themselves, ‘how is this problem or mathematical topic like things I have studied before?’” (p. 65). The second aspect suggests that students should profit from the interconnection of math ideas. This aspect makes students understand that skills and concepts are intertwined. Thus, “students will not perceive mathematics as an arbitrary set of rules” (p. 65). The third aspect suggests that math can be connected with other subject areas and disciplines. For instance, the lower grades students can connect math with the real world. Students in grades 3-5 can connect math with other subjects. Students in grades 6-12 can apply math to explain complex ideas in the outside world.

### Representation

Representation in the process standard includes three aspects:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representation to model and interpret physical, social, and mathematical phenomena. (p. 67)

The first aspect claims that both formal representations and idiosyncratic representations are very important for student learning. Students should have opportunities to construct, refine, and use their own representations and to learn conventional forms of representation as well. Representations help students reflect and organize their mathematical ideas in a concrete way. The use of computers and calculators results in an expansion of the set of representations. The second aspect emphasizes using multiple representations can help students understand different aspects of a complex concept or relations. The third aspect states that “in some activities, models allow a view of a real-world phenomenon, such as the flow of traffic, through an analytic structure imposed on it” (p. 70). Through K-12 grades, students should gradually learn to use representations to model a real-world phenomenon.

#### Overview of *Math Curriculum Standards for 1-9 Compulsive Education*

##### A Brief Introduction to Chinese Math Reform before 2001

It is difficult to provide a thorough review on the history of Chinese math educational reform in this section. The focus of this chapter is the math standards and principles in China and the USA. So here I give a specific review on the development of math standards in China's history. During the past one-hundred years, seventy-six math standards have been developed for elementary and secondary education by Chinese governments. Among these math standards, thirty-five of them were established before 1949, while forty-one of them were completed after 1949. Tracing back to these historical events, I conclude that the process of developing these standards is the process of adopting and assimilating external cultures. For instance, the standards of 1904 only recommended the concrete subjects for different grades. Schools had flexible choices to select content. The textbooks used in schools were the translations of textbooks from the USA, Japan, and the UK. After eight years of assimilation, the Chinese government enacted updated math standards and started to use the textbooks written by Chinese scholars in 1912. Between 1912 to 1946, the math standards were revised in a concrete and

specific way. That is, the teaching objectives became clearer; the difficulty of math content based on the teaching feedback had been considered when making new standards.

In 1949, the Chinese Ministry of Education enacted two math standards for elementary and secondary education respectively. These two standards were perceived as an elaboration of the previous math standards. The purpose of the elaboration and simplification was to keep the textbooks coherent for the effective teaching and learning. It was also recommended to make math knowledge applicable: one application in physics and chemistry, and the another application in economic development. Some abstract math contents that students had difficulty understanding were simplified or deleted.

However, perspectives from the former Soviet Union were adapted to the math standards from 1952 to 1963. For example, in 1952, the textbooks, such as algebra and trigonometry, were Chinese translations of the textbooks from the former Soviet Union. Arithmetic and geometry were mainly based on the former Soviet Union's textbooks. After many revisions, the Chinese Ministry of Education initiated its 1963 math standards with emphasis on computation ability, spatial visualization ability, and logical reasoning ability. These emphases were the results of cultural assimilation. Logical reasoning, rigor of proof, and basic knowledge were featured characteristics of math education in the former Soviet Union.

From 1978 to 1996, the updated math standards mainly focused on the change of the math content in different grade levels. Accordingly, the new textbooks were written based on latest version of math standards. For instance, in 1978, a serial of the mixed math textbooks were published for middle schools and high schools. In 1981, the mixed math textbooks for middle schools were replaced by algebra and geometry. In 1983, two kinds of math textbooks for high school were written for the different group of students based on the latest math standards. One type of the textbooks (Yizhongben) was at basic level, whereas the other type of textbooks (Jiazhongben) was at the advanced level. In 1988, eight regional textbooks were written based on

the 1988 math standards. Students in different regions were allowed to use different types of textbooks. During this period, in addition to “the three abilities” in the previous math standards, gradually developing students’ ability to analyze problems and solving problems was added to the teaching objective in the math standards. Meanwhile, Polya’s problem-solving and the ideas of NCTM’s 1980 agenda had been introduced to Chinese math community during that time.

Overview of the Chinese Ministry of Education 2001 *Math Curriculum Standards for 1-9 Compulsive Education* (MOE 2001 Math Standards)

The MOE 2001 math standards incorporate four parts: essential beliefs and the explanation of its design, the objectives of math curriculum, the content standards, and suggestions for implementing curriculum. This review focuses on the first two parts the essential beliefs and the objectives in the standards.

Six essential beliefs in the MOE math Standards serve as a foundation for math teaching and learning. The first belief is that math curriculum must be designed for all students. That is, everyone should learn valuable mathematics, everyone should obtain math knowledge necessary for their life, and different students should have different developments in mathematics. The second belief claims what math is: mathematics is perceived as a tool that helps people deal with data and calculating, reasoning, and proof. Mathematics models can describe both natural and social phenomena. Mathematics provides language and methods for science. Mathematics not only plays an important role on enhancing human reasoning, abstraction, and imagination, but also an important part of culture in modern civilization.

The third belief concerns mathematics learning. Two aspects are highlighted: learning task and learning activity. Students should learn mathematics contents that are meaningful, challenging, and real-life related. These contents must benefit students’ activities such as observing, experimenting, guessing, checking, reasoning, and communicating. Math content should be presented in different ways in order to meet the different needs. Effective learning activity does not rely on imitation and memorization. Rather, manipulatives, explorations, and

communications are the important ways for students to learn mathematics. Students' learning activity should be a vivid, self-motivated, and personalizing process.

The fourth belief is about teaching mathematics. Mathematics teaching must build upon students' prior experience and the levels of their cognitive development. Math teachers should motivate students to develop good attitudes toward mathematics. Math teachers also should give students opportunities for their own math learning. Math teachers also should facilitate students' self-exploration and communication in order to help them master basic knowledge and skills, math methods, and math learning experience. Math teachers should serve as organizers, facilitators, and collaborators.

The fifth belief focuses on assessment. The purpose of the assessment is to know students' learning processes, to encourage students' learning, and to improve teachers' teaching. The objectives and methods of the assessment should be multiple. The assessment of students' learning should not only focus on the students' achievement, but also focus on students' learning processes. Both levels of students' achievement and their emotions and attitudes should be observed.

The sixth belief regards technology. New technologies should be applied in the design and implementation of the new math curricula. Calculator and computer should be considered to integrate into the content of the math courses and the methods of student learning. Technology will serve as a powerful tool for students' problem solving. Technology makes learning efficient and, as a result, students have more time to participate in the real-life and exploratory math activities.

Aside from six beliefs listed in the MOE 2001 math standards, overall objectives are explicitly stated in the standards. The overall objectives break into four categories: knowledge and skill, mathematical thinking, problem solving, and effects and attitudes. These categories serve as a framework for the content standards in Chapter three. That is, the four dimensions of

math content, namely, number and algebra, shape and space, statistics and probability, and practical and synthetic application, are described in a concrete way in each category. Since our focus is to understand the beliefs and values in these math standards, the following review will provide a detailed description on the overall objectives.

Knowledge and skill, the first category in the overall objectives, are described in three aspects:

- Experience the process of abstraction from concrete or real life problem to numerical and algebraic problem; comprehend basic skills and basic knowledge of number and algebra, and solve simple problems.
- Experience the exploration of transformation and the relations of shapes, measurement, and positions on substance and graph. Comprehend basic skills and basic knowledge of space and shapes and solve simple problems.
- Experience the process of problem posing, data collecting and interpreting, decision making and predicting, comprehend basic knowledge and basic skills of statistics and probability, and solve simple problems.

Mathematical thinking, in the second category in the overall objectives, is encompassed of four aspects:

- Experience the process of describing real world phenomenon by using math symbols and graph, establish basic number sense and symbolic sense, develop abstract thinking.
- Establish basic spatial concepts, develop students' ability on spatial visualization.
- Experience the process by using data to describe information and to make conclusion, develop statistical thinking disposition.
- Experience the process of observation, experiment, guess, and proof in math activity, develop plausible reasoning and basic deductive reasoning, can state their own perspectives clearly and logically.

Problem solving, in the third category in the overall objectives, includes four aspects:

- Can basically pose and understand problems from mathematical perspectives, can use students' prior knowledge and skills for problem solving, developing their dispositions toward applying mathematics.



- Possess basic strategies for problem solving, experience multiple ways to solve problems, develop students' creativity and the ability to apply their knowledge in the real life.
- Be able to cooperate with others, and be able to exchange their thinking process and results.
- Develop meta-cognitive dispositions.

Affects and attitudes, in the last category in the overall objectives, comprise four aspects:

- Be able to actively participate in math learning activity, be curious about math, and desire to learn mathematics.
- Experience success in math activity, establish confidence, and be willing to overcome the hardships.
- Recognize the relationships between mathematics and human's life, experience exploration and creativity in math activity, feel the certainty of math results and the logical rigor of mathematics.
- Develop questioning dispositions and thinking independently.

Specific objectives for the three stages (grades 1-3, grades 4-6, and grades 7-9) are explained in detail after the overall objectives. The expected requirements are gradually advanced as the grades increase. For example, in the problem solving category, students in the first stage (grades 1-3) are expected to gain experience in cooperation with their peers. Students in the second stage (grades 4-6) are expected to be able to cooperate with others in a problem solving activity. Students in the third stage (grades 7-9) are not only able to cooperate with others, but also be aware of the importance of this cooperation.

Teaching suggestions for each stage are briefly stated in chapter four of the standards (p. 51; p. 64; p. 80). Three teaching beliefs are repeatedly claimed in each stage. The first one is that mathematics teaching is the teaching with math learning activities where teachers and students interact with each other in the process of co-developments. The second one is that teachers should create a learning environment where students can explore knowledge based on their prior experience or their real life experience. The third one is that the teacher should serve as an

organizer, a facilitator, and a collaborator in the class. Teachers should design teaching processes based on students' actual levels of knowledge and experience. Different students should have different developments in mathematics.

### Similarities and Differences between the Two Math Standards

I summarize eight similarities and two differences between the NCTM 2000 and MOE 2001 standards. This summarization does not consider the comparison of the requirements of the math content in different grades in each country. I am mainly concerned with beliefs and values regarding teaching and learning, and the important change of content and technology (e.g., statistics and probability, calculator) in both math standards.

#### Similarities

- 1) Both the NCTM 2000 and MOE 2001 standards advocate that teaching for understanding.
- 2) Both the NCTM 2000 and MOE 2001 standards claim teaching for all students, and different students should have different development in mathematics.
- 3) Both the NCTM 2000 and MOE 2001 standards emphasize that learning tasks should connect with real life and students' past experiences.
- 4) Both the NCTM 2000 and MOE 2001 standards state teachers' roles as an organizer, a facilitator, and collaborator.
- 5) Both the NCTM 2000 and MOE 2001 standards emphasize problem solving as an important aspect for math teaching and learning.
- 6) Both the NCTM 2000 and MOE 2001 standards claim that teaching tasks should be designed to meet students' needs.
- 7) Both the NCTM 2000 and MOE 2001 standards contend technology can enhance teaching and learning.

- 8) Both the NCTM 2000 and MOE 2001 standards state that statistics and probability are very important content for students' learning.

#### Differences

- 1) The NCTM 2000 standards is research-based, while the MOE 2001 math standards is practice-based. The NCTM 2000 math standards provide concrete principles and values on teaching and learning, whereas the MOE 2001 math standards state these principles and values in an abstract way.
- 2) The NCTM 2000 standards is a result of historical reflection, while the MOE 2001 math standards is a result of external cultural importation and assimilation.

#### Discussion

The NCTM 2000 and MOE 2001 standards share the similar fundamental beliefs and values in terms of math teaching and learning. Both standards advocate constructivist beliefs with regard to teaching goals, learning principles, learning tasks, student-teacher relations, and the like. The differences between the two standards are mainly demonstrated in their own cultural roots.

The NCTM 2000 standards is based on research with more than 200 references. Hiebert (2003) confirmed, "The standards are consistent with the best and most recent evidence on teaching and learning mathematics" (p. 5). In contrast, the MOE 2001 math standards did not claim any research bases. This does not indicate that the MOE 2001 math standards are of poorer quality than the NCTM 2000 math standards. Rather, this indicates, on the one hand, the MOE 2001 standards is a result of adopting and assimilating Western cultures; on the other hand, a result of following a Chinese writing tradition—most Chinese math standards were written in an abstract way with a focus on the requirement of math content in the past 100 years. Nevertheless, the imported cultural beliefs (e.g., constructivist beliefs) become the predominant ones in the MOE 2001 standards. The NCTM 2000 standards may be perceived as an extension of NCTM

1989 standards and 1980's *an agenda for action*; a continuing effort to get rid of the influence of behaviorism and 1960's the New Math Movement with regard to teaching and learning.

The similar beliefs and values of the NCTM 2000 and MOE 2001 standards arrange in different ways. Six principles and five process standards constitute the main beliefs and values in teaching and learning in the NCTM 2000 math standards. Similar to six principles in the NCTM 2000 standards, the MOE 2001 standards start with six beliefs. However, the MOE 2001 standards set up an overall objective and specific objective for students' learning. Although these objectives and suggestions are almost equivalent to the five process standards and a part of six principles in the NCTM 2000 standards, the new arrangements of these beliefs and values reflect a cultural assimilation and some features of Chinese way to teach and learn math. The preference to use category to state math teaching and learning might be influenced by Bloom's taxonomy, a theory very popular in Chinese education community. One of the four categories in overall objectives is knowledge and skills, with emphasis on comprehending basic knowledge and basic skills. Recent year Chinese math scholars have identified this emphasis as an important feature of Chinese ways for math teaching and learning.

The NCTM 2000 Standards is an elaboration and an update of its 1989 Standards, while the MOE 2001 standards can be perceived as a political force on Chinese education reform. High-speed economic development in China and the wave of international globalization catalyzed unprecedented Chinese elementary and secondary educational reform in 2000. Under this reform background, the MOE 2001 standards aimed to establish new math curricula and to change teachers' teaching beliefs. An implicit augment here is that Chinese ways of teaching and learning before 2000 were denied by the new educational reform.

However, many comparative researchers (e.g., Fan, Cai, Wong, & Li 2004; Zhang, 2004) conducted tremendous research to demonstrate Chinese ways of teaching and learning, and claimed the uniqueness and effectiveness of math teaching and learning in China. This might be

a new paradox regarding Chinese math education (note: the old paradox regarding Chinese ways to teach and learning have been thoroughly discussed by Chinese scholars in Hong Kong).

## Culture Models and Indigenous Culture Context of Teaching and Learning

Culture shapes mind, that it provides us with the toolkit by which we construct not only our worlds but our very conceptions of our selves and our powers. (Bruner, 1996, p. x)

Cross-cultural comparisons can help us discover characteristics of our own culture that we fail to notice because we are so familiar with them. (Stevenson & Stigler, 1992, p. 16)

In recent years, along with the thriving of international education comparisons and globalization, culture and culture-related teaching and learning have attracted much attention from comparative researchers. In terms of cultural influence in comparative education, many researchers in comparative education tend to treat culture as a factor or a variable to explain educational phenomenon in different countries (e.g., Cao, Bishop, & Forgasz, 2007; Papanastasiou, 2002; Su, Hawkins, Huang, & Zhao, 2001; Liu, & Teddlie, 2005, 2007). Instead of considering culture (or indigenous culture) as a prior cultural construct, these researchers usually start their investigations in a specific domain, and then use cultural factors to explain of certain phenomenon. Indeed, Culture is loosely defined in these studies.

Many comparative researchers have concluded that developing cultural models, both universal culture models and indigenous culture models, is essential for more deeply understanding cultural influence. Li (2002/2003), Stevenson and Stigler (1992), Stigler and Hiebert (1999), and Wong (2004) have made efforts to establish indigenous culture models to understand learning and teaching in the local context. Here indigenous culture refers to local culture (e.g., national or regional culture).

On the other hand, universal cultural models are thriving outside comparative education—cultural dimension(s) (e.g., Individualism/Collectivism Construct, Hofstede's cultural dimensions) have been investigated and applied widely since Hofstede's 1980 seminal work. In this review, the important cultural issues will be reviewed, clarified and synthesized in

three aspects: 1) The models of culture in comparative study, including universal culture models and indigenous culture models, 2) Teaching and learning in the indigenous cultural contexts, China and the USA, and 3) Discussions of the culture models based on two-level culture analysis in comparative education.

### The Models of Culture in Comparative Study

There are only a few culture models developed and used in comparative education. I classify cultural models in two categories: universal cultural models and indigenous cultural models. The universal cultural model refers to cultural dimensions, whereas the indigenous cultural model refers to a national or regional cultural construct. In this section, I start by reviewing universal cultural models from the other fields (e.g., business management, psychology) because of the paucity of the culture models in education. These models have been revised or applied for comparative education. In contrast, indigenous cultural models have directly focused on teaching and learning.

#### Universal Cultural Models

Universal cultural models have been prevailing in comparative study since 1950s (e.g., Kluckhohn, 1951; Parsons & Shils, 1951; Hofstede, 1980; Walker & Dimmock, 1999). A common approach used by these researchers is to establish cultural dimensions. Among these researchers, Hofstede's work had a significant impact on comparative study—it was perceived “as the most influential in the field of international comparative management over the last eighteen years” (Walker & Dimmock, 1999, p. 326). Hofstede's cultural dimensions influence researchers in comparative education in two ways. One is to create the new cultural dimensions for cultural analysis. For example, Walker and Dimmock (1999) developed six cultural dimensions for educational leadership research. The other one is that researchers use Hofstede's dimension directly in comparative education. In this section, Hofstede's and Walker and Dimmock's cultural dimensions are mainly introduced.

Hofstede (1980) identified four cultural dimensions confirmed through statistical analysis. Over 116,000 questionnaires from 40 countries were collected for this study. The four dimensions were labeled as Power Distance, Uncertainty Avoidance, Individualism, and Masculinity. Hofstede defined Power Distance as:

The power distance between a boss B and a subordinate S in a hierarchy is the difference between the extent to which B can determine the behavior of S and the extent to which S can determine the behavior of B. (Hofstede, 1980, p. 99)

This concept is rooted in the inequality in society and the inequality in organizations. Hofstede's work was influenced by Mulder and Kipnis who made contributions on the concept of power distance research in the 1970s.

Uncertainty Avoidance refers to the degree of the tolerance for uncertainty and ambiguity. Hofstede claimed that this factor led some individuals in the same situation to perceive a greater need for action for overcoming uncertainty than others (p. 161). Individualism indicates the relationship between the individual and the collectivity. The key feature of an individualist society is that the relationships among individuals are loose—the primary concern for a person is him/her self rather than family, group, or society.

Masculinity refers to that “The duality of the sexes is a fundamental fact with which different societies cope in different ways; the issue is whether the biological differences between the sexes should or should not have implications for their roles in social activities” (Hofstede, 1980, p. 261).

An assumption for having the universal cultural model is the idea of a universal level of human mental programming. Hofstede argued,

We can distinguish broadly three levels of uniqueness in mental programs, ... The least unique but most basic is the *universal* level of mental programming which is shared by all, or almost all, mankind. This is the biological “operating system” of the human body, but it includes a range of expressive behaviors such as laughing and weeping and associative and aggressive behaviors which are also found in higher animals. (p. 15)

Hofstede's cultural model was widely used in comparative study (e.g., The Chinese Culture Connection, 1987; Hui & Villareal, 1989; Schwartz, 1990; Trafimow, Triandis, & Goto, 1991; Walker & Dimmock, 1999; Bochner & Hesketh, 1994; Niehoff et al., 2001; Nguyen, Terlouw, & Pilot, 2006). Quantitative researchers tend to validate the constructs and to explore the antecedents and consequences, while qualitative researchers intend to test various implications of Hofstede's model. Among these researchers, Walker and Dimmock conducted research on educational leadership and established new cultural dimensions based on Hofstede's and others' cultural models. They first reviewed existing frameworks, and then compared the differences and similarities between the dimensions in the various frameworks. As a result, the useful parts in the existing models were adapted to their new model for educational leadership study. They finally regrouped and relabeled six dimensions as the foundation of a cross-cultural comparative framework.

The first dimension in Walker and Dimmock's model is *Power-distributed/power-concentrated*. Walker and Dimmock (1999) claimed that "Power is either distributed more evenly among the various levels of a culture or highly concentrated among the few" (p. 333). So they believed the relabeled dimension more accurately captured the essence of power relationships than Hofstede's power distance dimension.

The second dimension in Walker and Dimmock's model is called *Group-oriented/self-oriented*. This dimension describes the degree to which people are integrated into groups. Individualism/communitarianism (Trompenaars & Hampden-Turner, 1997) and individualism/collectivism (Hofstede, 1991) were adopted to this dimension.

The third dimension in Walker and Dimmock's model is *Consideration/aggression*. This dimension is the reconceptualization of Hofstede's masculinity dimension. The meanings in this dimension is not restricted to gender issues. In aggression cultures, "achievement is stressed, competition dominates, and conflicts are resolved through the exercise of power and



assertiveness” (Walker & Dimmock, 1999, p. 334). In contrast, consideration cultures go to another direction, “emphasis is on relationship, solidarity, and resolution of conflicts by compromise and negotiation” (p. 334).

Walker and Dimmock named the fourth dimension as *Proactive/fatalistic*. This dimension derives from three sources: Trompenaars and Hampden-Turner’s “attitudes toward the environment” category, Hofstede’s “Uncertainty Avoidance” dimension, and Walker and Dimmock’s thinking of the concepts of “opportunistic” and pragmatism/idealism. In proactive cultures, people tend to believe they make their own luck. By contrast, people in fatalistic cultures tend to believe “what is meant to be, will be” (p. 335).

The fifth dimension in Walker and Dimmock’s model is *Generative/replicative*. This dimension addresses that people in generative cultures incline to value new ideas and generation of new knowledge. On the other hand, people in replicative cultures tend to replicate ideas and methods from elsewhere. This dimension is not drawn from Hofstede’s model.

The sixth and final dimension in Walk and Dimmock’s model is *Limited relationship/holistic relationship*. Trompenaars and Hampden-Turner’s “Specific/Diffuse” and “Performance/Connection” categories and Walk and Dimmock’s previous work are adopted to build this dimension. People in limited relationship cultures tend to abide firm rules, while people in holistic cultures emphasize relationship obligations.

The similarities and differences between Hofstede’s cultural dimension and the work of Walk and Dimmock can be summarized as follows:

- Both Hofstede and Walk & Dimmock agreed that developing cultural dimensions is powerful for comparative study.
- Hofstede’s work was for international business comparison, while Walk and Dimmock’s work was for educational leadership comparison.

- Hofstede's work mainly derived from the empirical data. In contrast, Walk and Dimmock's work was mainly drawn from theoretical literature and their own practice.
- Both Hofstede and Walk & Dimmock claimed that they did not hold a static view on culture. However, they did not provide a pragmatic solution between the static category and the dynamic cultural elements.

### Indigenous Cultural Models

In contrast with universalists such as Hofstede, Walker, and Dimmock, some researchers (Li, 2002, 2003; Wong, 2004; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999) focused their efforts on establishing indigenous cultural models. Unlike universal culture models, most of the indigenous culture models introduced here have a direct focus on learning and teaching.

Li (2002) presented a Chinese learning model called “Heart and mind for wanting learning”—the Chinese term *hao-xue-xin* (HXX). Two groups of Chinese college students, 122 in total, were selected for this research. The students in the first group were asked to describe a real person who was an ideal learner from his/her perspective. The students in the second group were allowed to sort the features that were identified in the first group's responses. Li finally constructed HXX in four components: cognitive-conceptual ideal of HXX, moral-virtuous ideal of HXX, behavioral ideal of HXX, and affective ideal of HXX. These components deeply reflect a Chinese learning tradition. These perceptions are consistent with Confucian learning beliefs.

One of the significant contributions in Li's work is that her study provides convincing evidences that Chinese traditional learning beliefs still exist in present society after the political storm of Culture Revolution in the 1960s and the 1970s. She first made this assumption, and then she proved it is true. Following her HXX model, Li's new investigation (2003) has extended culture understanding of learning in Chinese college students and American college students as well. She first asked participants to generate an English and a Chinese list of learning-related

terms. Then the participants were asked to sort the cards into groups based on their judgments of the similarity in meanings. Li (2003) concluded, “the present study showed much more complexity, with the U.S. beliefs elaborating on learning processes, learner characteristics, and related social context, and the Chinese beliefs primarily focusing not on intelligence but on other dimensions of life such as the personal, the social and the moral” (p. 265).

Table 2.1: Components and Dimensions of Chinese and US Beliefs about Learning

Chinese	US
<b>Purpose of Learning</b>	
Perfect oneself morally	Cultivate the mind/understand the world
Acquire knowledge/skills for self	Develop one’s ability/skill
Contribute to society	Reach personal goals
Obtain social respect/mobility	
<b>Process of Learning</b>	
Resolve	Active learning
Diligence	Thinking
Endurance of hardship	Inquiry
Perseverance	Task management
Concentration	Communication
(Virtue-oriented)	(task-oriented)
<b>Kind of Achievement</b>	
Breadth-depth/mastery of knowledge	Understanding of essentials/expertise
Application of knowledge	Personal insights/creative problem solving
Unity of knowledge and moral character	Being the best one can be
<b>Affect</b>	
Positive	
Commitment (“establish one’s will”)	Curiosity/interest/motivation
Love/passion/thirst (may not favor intrinsic source, but cultivated affect)	Intrinsic enjoyment
Respect	Challenging attitudes
Humility for achievement	Pride for achievement
Negative	
Lack of desire	Indifference/boredom
Arrogance	Extrinsic motivation
Shame/guilt for failure	Low self-esteem for failure

In her 2004 study, Li elaborated her previous findings into four dimensions with concrete components in each dimension (p. 138). Li summarized that American people demonstrated

“mind orientation” toward learning. That is, they perceived knowledge as a neutral body and concerned with cognitive skill, intelligence, and abilities. They also perceived learning as a process which was facilitated by motivational factors such as interest, curiosity, willingness, and commitment. Some items such as Cultivate the mind, Develop one’s ability/skill, Reach personal goals, Being the best one can be, and Personal insights, listed in Table 2.1, reflect an individualist orientation in the students’ minds.

Wong (2004) reviewed Confucian Heritage Culture (CHC) of teaching and learning and presented his perception of the CHC classroom environment and “CHC script” (p. 503). In recent years, Confucian Heritage Culture has been often used in comparative education to refer to Confucianism, Daoism, and Buddhism. So CHC learners are not restricted to those in mainland China. Japan, Singapore, Malaysia, Hong Kong, and Taiwan are also classified as part of the CHC tradition. Wong summarized eight descriptive perceptions of the CHC classroom environments:

- Obedient and attentive students sitting properly listening to the teacher.
- Teachers with their lessons well-prepared and structured.
- Students seldom interrupting the flow of the teaching by asking questions.
- Teachers checking whether students follow through by asking questions.
- Teachers not attempting to cater for individual differences in class yet.
- Students having a lot of guided after-class learning (including homework and tutorial classes).
- Teachers giving individual guidance after class.
- Teachers seeing the moral responsibility of providing individual care, including those not directly related to learning (e.g., personal growth and transmission of cultural values such as listenership). (p. 525)

Wong (2004) claimed that the above classroom teaching picture was in accord with Ausubel’s (1961, 1963, 1988a, 1968b) arguments that the teaching format can be “both teacher

led and student centered” (p. 526). Further, Wong proposed a possible CHC script (p. 527) as follows:

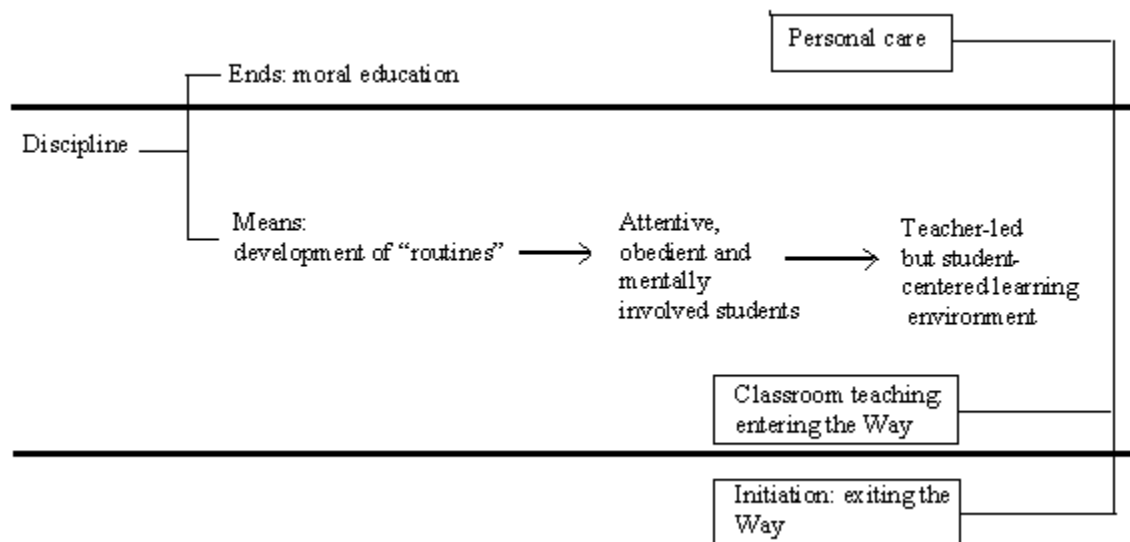


Figure 2.2: A CHC Script

In the above figure, moral education is informed by Confucian philosophy. The development of “routines” means the flow in classroom teaching: “when to talk, when to do seat work, when to open one’s book, when to look at the chalk-board (or computer projection), and so on” (p. 526). Entering the Way refers to the basic knowledge to be acquired by teachers’ instruction. Exiting the Way means “looking similar but being different” (p. 518) that could happen after the individual guidance.

Stevenson and Stigler (1992) explicitly posed cultural models of learning when comparing elementary students’ learning in China, Japan, and the USA. In their work, an ability model refers to American learning belief; in contrast, the effort model refers to Chinese and Japanese learning belief. The authors pointed out that Americans have been enthusiastic to test children’s innate ability for many decades. Measuring intelligence results in teachers reducing their expectations of the dull children. By contrast, Chinese and Japanese emphasize on effort more than inborn ability. This was caused by Confucian belief with regard to learning. Stevenson

and Stigler claimed that “Lack of achievement, therefore, is attributed to insufficient effort rather than to a lack of ability or to personal or environmental obstacles” (p. 98). The ability versus effort model seems to focus only on single cultural element analysis in different cultures. In fact, Stevenson and Stigler also picked up other cultural elements (e.g., satisfactions and expectations) to make comparisons in their book *The Learning Gap*.

Stigler and Hiebert (1999) presented a clear statement of an American teaching model—Behaviorist Teaching—by reviewing and analyzing 1999 TIMSS video study. Behaviorism has over one hundred years of influence in U.S. education. Behaviorist Teaching has become a fundamental belief among American teachers. In addition to the video observations, the interview data from Stigler and Hiebert’s study strongly supported their assertion. For instance, U.S. teachers believed that school math is a set of procedures, and sixty-one percent of U.S. teachers believed that their students learn *skills* in their lessons (p. 89). Under such beliefs, U.S. teachers tended to help students master procedures piece by piece. The roles of the teacher are perceived as “shaping the task into pieces that are manageable for most students, providing all the information needed to complete the task and assigning plenty of practice” (p. 92).

#### Teaching and Learning in the Indigenous Culture Contexts: China and the USA

##### Math Learning and Teaching in Chinese Culture Context

Before the 2001 math curriculum reform, Chinese classroom teaching emphasized “two basics” and “three types of abilities.” Two basics refer to basic knowledge and basic skills. Yet three types of abilities indicate computation ability, spatial visualization ability, and logical reasoning ability. Under these emphases, Chinese teaching can be summarized by the following six aspects (Zhang, Li, & Tang, 2004):

- 1) Teachers play a central role in classroom.
- 2) Effective teaching is emphasized.

- 3) The pattern of “teach only the essential and ensure plenty of practice” is used in teaching.
- 4) There is a distinctive way of mathematics teaching called “teaching with variation”
- 5) Mathematics teachers in China usually believe in the maxim that “mathematics is gymnastics of thinking”
- 6) Under the “Two Basics” principle, logical deductive reasoning is thought as the core of thinking ability.

The above six aspects demonstrate the uniqueness of Chinese teaching and the historical change from 1960 to 2000. In a classroom with 40-50 students, it seems natural for a teacher to play a central role. He/she must decide the time period for certain content and make sure if most students understand what he/she teaches. Effective teaching is usually connected with lesson plan implementations. If a teacher completes the teaching objectives in the lesson plan, his/her teaching is perceived as effective. Instead of discovery, group discussion, and real-life mathematics, questioning becomes a common strategy for testing students’ understanding and sparking their thinking. The practice issue in the third aspect reflects teachers’ beliefs on understanding. Chinese teachers believe that the understanding cannot be reached in a short time period. Sufficient exercises are necessary for students to consolidate knowledge and lead to understand concepts. “Teaching with variation” refers to a strategy that focuses on the procedures or forms by which the problems are presented in classroom. To some extent, this strategy provides a way for designing effective tasks in classroom teaching. The fifth aspect reflected how “two basics” and higher order thinking interplayed in the 1990s:

Since the 1990s, with the effort of numerous mathematics teachers, the “Two Basics” principle has been used to teach higher level of thinking in mathematics teaching. Curriculum standards set a higher level of teaching objectives in this aspect. Students need to master many “basic ways of thinking” flexibly, such as classification without overlapping, four types of prepositions and the conversions among them, necessary and sufficient conditions, induction and deduction, analysis and synthesis and mathematical method of reversion-mapping-inversion, etc. (Zhang, Li, & Tang, 2004, p. 197)

It is obvious that the culture elements interplayed and shaped Chinese teachers' beliefs and values. Here I briefly list these cultural elements with corresponding teaching and learning phenomenon. A collective cultural tradition (e.g., hierarchical social structure and obedience to authority) made the big classroom size possible in China. A balance between teachers' lecture and students' practice reflects a Taoist tradition. Chinese teachers' perspective on students' exercise, *practice makes perfect*, is rooted in Confucian learning tradition. Teaching with variation is also toward a Taoist tenet "Changing." On the other hand, imported cultural elements played an important role in shaping this Chinese teaching culture. For example, emphasizing deductive reasoning and mathematics foundation are the results of assimilating learning culture from the former Soviet Union. Emphasizing higher level thinking may be influenced by Polya's Problem-Solving and the ideas of NCTM 1980's Agenda and NCTM 1989 standards.

A unique characteristic in teaching from 1960 to 2000 in China was the innovative teaching strategies created by the model teachers or by the influential educational experiments. This was rarely mentioned by comparative researchers when they articulated Chinese ways of teaching and learning. These teaching strategies were usually a recreation based on or partly based on the above six teaching aspects. That is, the big classroom size was rarely changed; the teachers' role was rarely changed dramatically; the changes usually occurred when adopting new culture elements (e.g., problem solving ability), or implicitly integrating Chinese fundamental beliefs into the curriculum. For instance, Gu, a math educator in Qingpu County, Shanghai, and now a professor, started his math teaching experiment in 1977. He summarized the following strategies for teaching—that became one of the most influential examples in China.

1. Using problems as a starting point for teaching.
2. Guiding students to develop exploratory activities.
3. Establishing variation in practice to raise the effectiveness of practice.
4. Summarizing to adopt into the knowledge structure, and



5. Modifying according to the fine categorization of teaching objectives. (cited in Lopez-Real, Mok, Leung, and Marton, 2004)

Like Gu, many Chinese teachers (e.g., Weigang Sun, Qianxiang Zhao, Guimei Dou, Shusheng Wei) earned nationwide reputations in terms of their teaching experiments. Many teachers are eager to emulate these model teachers' teaching. In China, learning a popular teaching experiment is highly supported by the school boards or education administrators. Emulating these experiments and teaching strategies truly reflected a Confucian tradition—a social cultural property that is implicitly embedded in the Chinese cultural context.

#### Math Teaching and Learning in the U.S. Culture Context

Featured as a de-centered education system, the U.S. math education reforms have been debated hotly nearly 100 years. From early in the 20<sup>th</sup> century until the 1950s, American math education was dominated by progressive education beliefs along with the big influence of behaviorism. From 1950 to 1970, the New Math Movement was prevailing in the U.S. Nevertheless, there was no coherent philosophical claim in the New Math Movement. In the 1970s, a new slogan “back to the basics” was advocated because of the failure of the New Math Movement. From 1980 to 2000, NCTM's series of efforts became influential in math education reform. The philosophical foundation of NCTM's math standards (1989, 2000) was consistent with constructivist beliefs and a part of progressive education beliefs.

Progressive education beliefs can be traced back to John Dewey. In math education, progressivist William's claims became the most influential around the 1920s. His book, *Foundations of Method*, was used as a standard textbook for teacher education across the country in 1925. Another remarkable work written by William was the report *The Problem of Mathematics in Secondary Education*, which was published in 1920 by the U.S. Commissioner of Education, Philander P. Claxton, a friend of William. Klein (2203) summarized William's views of point as follows:

Reflecting mainstream views of progressive education, Kilpatrick rejected the notion that the study of mathematics contributed to mental discipline. His view was that subjects should be taught to students based on their direct practical value, or if students independently wanted to learn those subjects. This point of view toward education comported well with the pedagogical methods endorsed by progressive education. Limiting education primarily to utilitarian skills sharply limited academic content, and this helped to justify the slow pace of student centered, discovery learning, the centerpiece of progressivism.

During this period, progressivists also sought support from behaviorist learning theory (e.g., Thorndike's stimulus-response). However, perspectives from progressivists and behaviorist with regard to teaching and learning are not a monolithic voice in math education. In 1920, the NCTM was established in part to counter the progressivist's agenda in math education. In 1923, NCTM disseminated the report, *The Reorganization of Mathematics for Secondary Education*, which was written by mathematicians and prominent teachers. This report included a survey of math curricula, the training of math teachers in other countries, issues of math learning related to psychology, and the intrinsic value and application of mathematics. It is also claimed that algebra was very important for every student, a view that was opposite to William.

Progressivism strongly influenced math education in the USA during the 1920s and the 1940s, although it was criticized by mathematicians and others. In the 1950s and the 1960s, the New Math Movement replaced progressive math. Mathematicians were highly involved in this movement. Unlike progressive math, curriculum in the New Math Movement emphasized coherent logical explanations. Calculus courses were introduced at the high school level. A fatal weakness of the New Math curriculum was that the math content was extremely formal. Set theory and exotic topics were introduced without considering students' previous experience. Basic knowledge and math applications were ignored. In the early 1970s, the New Math Movement was replaced by "back to the basics." In the 1980s, the NCTM published a remarkable report, *An Agenda for Action*. Problem solving was the heart of this report. From 1989 to 2000, the NCTM published serious standards documents, and these documents served as

guidance for math educational reform. The developers of the NCTM 1989 standards acquiesced that constructivism as underpinning foundation of the standards. However, this was an after-the-fact consideration and a set of beliefs, as Bosse (1995) commented:

The NCTM, therefore, while possibly beginning the work on the Standards with a fractionated philosophy, concluded with the entire group coming together under the banner of constructivism. Due to multiple definitions for Constructivism, however, stating that the NCTM has espoused Constructivism could be in itself naïve. There are three common variations of Constructivism: Constructivism as doing mathematics by abstracting, analyzing, proving, and applying; Constructivism as constructing cognitive models and representations; and Constructivism as the process of societal enculturation. So, to admit the Constructivist paradigm means, at best, to ascribe to one of at least three perspectives on Constructivism; this can hardly be equated with stating that all who subscribe to the tenets of Constructivism are unified in one philosophical, epistemological, position. (p. 87)

The NCTM 2000 math standards, an update of its 1989 standards, still held a set of constructivist beliefs. Some claims in the NCTM 1989 and 2000 standards also reflect progressivists' perspectives: for example, the claim that teaching should be based on students' past experiences; the claim that teaching materials should be real-life and situation-oriented; and the claim that teachers should serve as facilitators for students' learning. From 1980 to 2000, constructivism as a theory has been developed and applied in math education. A detailed description of this development can be found in the constructivism literature review in this chapter.

In the actual teaching practice, American teachers tend to adopt a behaviorist approach. Stigler and Hiebert (1999) analyzed 1999 TIMSS video-taped lessons from the USA, Japan, and Germany. The authors contended that teaching was a cultural activity, and the typical teaching patterns drawn from the American classroom were behaviorist. Four activities characterized U.S. lessons:

- Reviewing previous material. The lesson begins by checking homework or engaging in a warm-up activity.
- Demonstrating how to solve problems for the day. After homework is checked, the teacher introduces new material, or reviews previous material, by presenting a few

sample problems and demonstrating how to solve them. Often the teacher engages the students in a step-by-step demonstration by asking short-answer questions along the way.

- Practicing. Seatwork is assigned, and students are asked to complete problems similar to those for which the solution method was demonstrated. Seatwork usually is done individually.
- Correcting seatwork and assigning homework. Near the end of the lesson, some of the seatwork problems are checked and, occasionally, some additional problems are worked out together. Homework, with more practice problems, is then assigned. (p. 81)

These activities are closely related to stimulus/response patterns—a behaviorist assumption of learning. American teachers' actions, such as checking and demonstrating, truly reflected behaviorist orientation in their teaching. A step-by-step demonstration also indicated that the teacher believed the students could imitate the teacher's work and learned knowledge incrementally.

#### Discussion of Cultural Influence on Comparative Education

In this part, the cultural models are discussed by using two-level cultural analysis, a core issue in sociology. Two-level cultural analysis refers to the debate in sociology regarding the relations between social phenomenon and individual actions and beliefs. In comparative study, this issue has puzzled comparative researchers for several decades. Before discussing this troublesome issue, I introduce two wonderings from Asian scholars. One is described in Wong's work, and the other is posed by Lopez-Real, Mok, Leung, and Marton.

The first wondering concerns who accounts for Confucian Heritage Culture. In recent years, Eastern countries and regions, such as China, Hong Kong, Taiwan, Japan, Singapore, Malaysia, and South Korea, are identified as Confucian Heritage Culture (CHC), loosely defined in terms of Confucianism, Taoism, and Buddhism. However, some of the countries and regions also are influenced by other cultures. Wong (2004) described the CHC disputes in a math education conference in Asia:

A participant from Singapore objected to the classification of Singapore as a CHC region on the grounds that Singapore was a country of multi-cultures. . . . We can also doubt whether Hong Kong, being greatly influenced by Western culture, could be classified as a CHC city...it is not easy to account for the case of Mainland China when traditional culture was once wiped out by Communism. There are 28 provinces and 56 ethnic groups in China. Geographically, it is not easy to identify central China. (p. 511)

The second wondering is whether a national script exists (Lopez-Real, Mok, Leung, and Marton, 2004, p. 382). In fact, these researchers did not believe a national script exists. They questioned the methodology used by Stigler and Hiebert in 1999 TIMSS video study. They wrote, “First, it is dangerous to attempt to identify a teacher’s ‘script’ from a selection of ‘snapshot’ lessons. Second, such a characterization may suggest implicitly that a teacher’s approach is almost invariable from lesson to lesson” (p. 382). Instead of seeking the national scripts, they tried to identify “a pattern of teaching” by observing one teacher’s consecutive lessons. They defined a “pattern of teaching” as “The identifiable features of a teacher’s classroom practice, occurring in a repeated manner over a period of time, that together constitute the characteristics of the teacher’s style” (p. 409).

To answer these two wonderings, we need to go to Sociology and Psychology to find out related culture issues—two-level culture analysis. This is, whether a social property can be reduced to individual properties, and whether the sum of the individual properties is equal to a social property. If one believes that higher-level properties exactly match lower-level properties, and vice versa, one holds a reductionist perspective. If one believes that higher-level properties cannot be explained by the lower-level properties, and the higher-level properties are dynamically self-sustained, one holds a holistic perspective (e.g., gestalt psychology). In sociology, researchers have developed a variety of theories and concepts to analyze the relationships between social properties and individual beliefs. For instance, emergence theory provides a non-reductive materialist perspective. However, these perspectives are rarely adapted

to comparative education when comparative researchers discuss cultural models or national scripts.

If we treat the Confucian Heritage Culture (CHC) as a national (or cross national) social property, the first wondering seems to question how to reduce the CHC construct into individual levels or sub-regional levels. For the second wondering, the authors seemed to doubt individual-level properties can make up a social property.

Researchers who advocated universal culture models did not deny levels of culture(s) in their study. Hofstede implicated two levels— individual and societal level—in his data analysis. Only societal level data were associated with the national culture dimension. Otherwise, conflicting statistical results might be obtained. For instance, individual-level data did not support his Individualism/Collectivism dimension. Dimmock (2000) claimed that “a comparative framework based on dimensions of societal-level culture needs to be complemented by an equivalent set of cultural dimensions at the organizational level” (p. 54). Consequently, the elaborated six dimensions of organizational culture were developed by Dimmock for the leadership study. Compared to Hofstede’s two-level analysis, Dimmock and Walker did not explicitly point out the relationship between individual-level culture and societal-level culture phenomenon—they paid more attention to the elaboration of Hofstede’s cultural dimensions. These researchers acknowledged the dynamic feature of culture(s). For instance, Walker and Dimmock (1999) addressed that “the application of the framework does, however, present a snapshot of cultural influences at a certain point in time” (p. 343).

However, it is difficult to free cultural elements from a static sense due to the use of universal dimensions and statistical methodology. Specifically, universal dimensions become problematic when directly applied to comparative education. For example, Niehoff, Turnley, Yen, and Sheu (2001) conducted a study exploring cultural differences in classroom expectations of students from the USA and Taiwan. In this research, Hofstede’s four cultural dimensions were

used as a prior culture construct. These kinds of applications have made Hofstede's cultural dimensions a societal-level static category.

Indigenous culture models (Li, 2002, 2003) demonstrated a societal-level structural category. Li adopted the same approach developed by Shaver and others. The underlying assumption of this approach is that people construct generic mental representations of categorical systems of objects or events by their repeated experience. These human mental representations of categorical systems can be structured in two dimensions: vertical and horizontal. Li gave detailed descriptions of these dimensions in her 2003 study:

The vertical consists of three main levels: superordinate, basic, and subordinate. Whereas the superordinate level concerns the broadest distinctions (e.g., furniture), the basic level contains less abstract differences (e.g., Chair). The subordinate level captures the finer categorical differences (e.g., kitchen chair). The horizontal dimension pertains to the categorical distinctions at the same level (Rosch, 1978). A characteristic of the horizontal dimension is that many common categories (e.g., chair, sofa) are "fuzzy sets," meaning that people classify by general resemblance, not precise definitions-thus, the term prototypes. (p. 258)

Li (2002) also mentioned that there was no one-to-one mirror correspondence between a given cultural model and the members of the culture (p. 251). This implies a non-reductionist perspective.

Providing a coherent perspective regarding the relationships between two-level cultural analysis is an important task in comparative education. This will resolve the two wonderings and other controversial issues. For instance, at the individual level, it is hard to find a teacher's lesson plan toward moral perfection as described in Wong's CHC script. However, at the national level, there was a clear tendency to imitate the model teachers and famous teaching experiments as I reviewed. A detailed analysis of a new cultural model, which emphasizes both levels in analysis and reveals the relations between the two-level cultural analysis, will be provided in Chapter Three.

## CHAPTER THREE: THEORETICAL MODELS AND DATA ANALYSIS

### A Model of Indigenous Culture of Learning and Teaching in China and the United States

This theoretical study aims to establish a cultural model of teaching and learning in China and the USA for the purpose of analyzing the influence of indigenous culture in comparative studies of teaching and learning in these two countries.

Researchers in comparative education are agreed that the purposes of comparative study can be articulated in two phrases, “to understand” and “to learn from” (Halls, 1990; Holmes, 1971; Lauwerys, 1969; Phillips & Schweisfurth, 2006). The goal of comparatively understanding some aspect of educational practice requires a model of the most influential beliefs and values regarding learning and teaching in the two countries. It is only within the past thirty years that comparative researchers noticed and warned that the national culture plays a fundamental role in the adoption of promising practices from other cultures. Within that period, a few influential cultural models have been developed for purposes of comparative education (e.g., Ho, 1991 cited in Wong, 2004; Hofstede, 1980; Li, 2003, 2004; Walker & Dimmock, 1999). Within my interest area of mathematics education, many recent studies have specifically factored culture-related issues in comparative analyses (e.g., Li, 2003, 2004; Phuong, Terlouw, & Pilot, 2006; Stigler & Hiebert, 2000; Thomas, 1997). The new model presented here is more specific than the extant cultural models of learning and teaching in China and the U.S.

#### Theoretical Framework

Emergence theory is adopted in our definition of culture in this study. Emergentists hold the view of non-reductive materialism. At the level of individual mental functioning, this position maintains that “mental properties are supervenient on the physical brain and yet not



reducible to physical properties” (Sawyer, 2001, p. 580). At the societal level, the parallel perspective is that “social properties are supervenient on individual properties and yet not reducible to those properties” (Sawyer, 2001, p. 580). From this perspective, I perceive indigenous culture as a collection of interconnected social properties realized in the collective activity of a cultural group. It is dynamically stable but subject to gradual change. Beliefs and values about knowledge are an important part of the indigenous culture of teaching and learning that are realized in the activities concerning schooling within a culture.

To further articulate the relationships between social properties and individual properties, I introduce two important concepts, supervenience and multiple realizability. Supervenience refers to the perspective that individual properties determine social properties. That is, if two events are identical at the individual levels, they cannot differ at the social levels. Multiple realizability is involved in the debate on the relations between a psychological term and some combination of neurobiological terms, as Sawyer (2001) stated:

[multiple realizability refers to] the observation that although each mental state must be supervenient on some physical state, each token instance of that mental state might be implemented, grounded, or realized by a different physical state. For example, the psychological term “pain” could be realized by a wide range of different neurobiological terms and concepts, and each token instance of “pain” might be realized by a different supervenience base. (p. 557)

The emergence account with supervenience and multiple realizability can help us define indigenous culture from a new angle. First, I perceive indigenous culture as a dynamic process between the two-level—social level and individual level—interactions. At the social level, cultural elements emanate from the historical influence; while at the individual level, cultural elements are acquired by individuals within the certain cultural context. The two-level cultural interactions constitute the whole indigenous culture. Secondly, cultural elements at the social level are supervenient on the cultural elements at the individual level. For instance, if two cultural elements are identical at the individual level, these elements must be identical at the

social level. However, it is not necessary to find a matching element at the individual level when I identify a cultural element at the social level. Thirdly, when seeking the connection between the social level and the individual level, one cultural element at the social level might be connected with different sources of cultural elements at the individual level—this refers to multiple realizability.

The following graph illustrates the relationships of cultural elements regarding the indigenous culture of learning and teaching. As a collection of interconnected social properties, culture is manifested as both cultural precepts or principles and cultural practices as shown below. Mutual influence constitutes the dynamic features of culture, as illustrated in Figure 3.1.

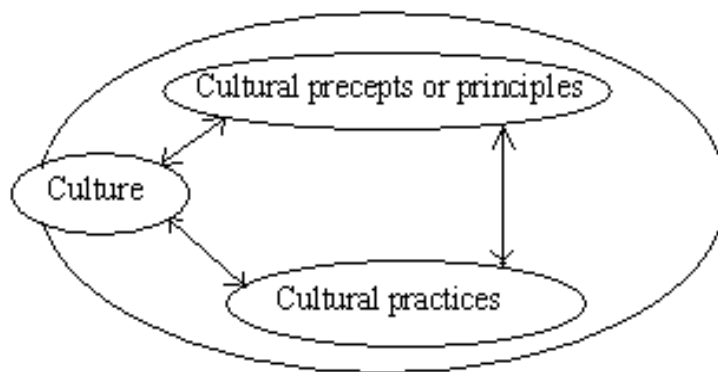


Figure 3.1: Culture and Practice

From the emergentist perspective, cultural precepts and principles are perceived as more stable social properties than cultural practices. Cultural practices are understood as current individual actual practices within a cultural context. This framework implies that one cannot explain individual actual practices completely based on cultural precepts or principles, nor can one fully deduce cultural precepts or principles from examination of individual practice; though the levels of analysis are deeply interdependent. This conception helps resolve conflicting perspectives in comparative research. For example, researchers have argued that Eastern countries and regions, such as China, Hong Kong, Taiwan, Japan, Singapore, Malaysia, and

South Korea, are identified as Confucian Heritage Culture (CHC), loosely defined in terms of Confucianism, Taoism, and Buddhism. However, some of the countries and regions also are influenced by other cultures. Wong (2004) described the CHC disputes in a math education conference in Asia:

A participant from Singapore objected to the classification of Singapore as a CHC region on the grounds that Singapore was a country of multi-cultures...I can also doubt whether Hong Kong, being greatly influenced by Western culture, could be classified as a CHC city...it is not easy to account for the case of Mainland China when traditional culture was once wiped out by Communism. There are 28 provinces and 56 ethnic groups in China. Geographically, it is not easy to identify central China. (p. 511)

The objections and questioning of CHC demonstrates the shortcomings of existing perspectives on indigenous culture. Researchers perceive CHC as a static and structural category. CHC was intended to MATCH with actual practices. From an emergentist perspective, perceiving culture as a collection of interconnected social properties one does not need to object to the existence of other culture influence. CHC can be identified as one of the culture properties, and it can interact with other culture properties in a certain area or country over time. It is not necessary to match CHC with actual practices too—CHC and actual practices can interact together with non-reductive features.

This emergent cultural conception provides an alternative, poststructural, way to establish cultural models that attends to possible interrelations of specific social properties and specific actual practices. In contrast, the extant cultural models tend toward a universalist representation of culture. Hofstede's (1980) model, for instance, contained four dimensions that were perceived as static categories that could be applied to all societies and nations. Although a universal model may be convenient for cultural analysis, it risks disconnection of the dimensions in the static category from actual cultural practices. This study seeks to establish a specific culture model for analyzing teaching and learning in China and the USA that is responsive to actual teaching practices.

## Analysis of the Indigenous Cultural Elements in the New Model

In the new model, Confucianism and Taoism emerge as the most influential beliefs and values in terms of teaching and learning in China, in contrast with Behaviorism and Individualism in the USA. According to the cultural conceptions in Figure 3.1, I am concerned with the two level interactions, Precepts or Principles and Cultural Practices, when selecting cultural elements in both countries. That is, I first trace back to the histories in both China and the USA in order to find prominent theories, beliefs, and values that have a long time influence in education. And then I examine if these theories, beliefs, and values are still reflected in current cultural practices in terms of teaching and learning.

### Analysis of Confucian and Taoist Cultural Elements

Notions of Confucianism adopted to this study reference the Confucian precepts and saying in *Analects*, a classical book that best represents Confucius' perspectives on teaching and learning. Taoist ideas adopted in this study reference the Taoist belief in "changing" and "balancing" as incorporated into *yin-yang* principles. Both *Analects* and *yin-yang* principles have heavily influenced Chinese culture with regard to learning and teaching over two thousand years.

Confucianism as dominating beliefs has permeated Chinese history for over 2000 years. It started from Confucius (Kongzi) and was elaborated and developed by his followers. As my interest is in teaching and learning, I only focus on analyzing beliefs and values from the *Analects* to see if these beliefs and values still influence Chinese teachers' thinking.

The *Analects* contains about 500 pieces of dialogues between Confucius and his disciples. This collection was edited by Confucius' disciples after his death. The *Analects* reflects Confucian perspectives on human self-cultivation, teaching and learning, administrative management, and the like. In this review I only introduce Confucian perspectives on teaching and learning. Confucius believed that students must cultivate good learning attitudes: enduring

hardships, overcoming the difficulties, and then enjoying learning. The following extracts illustrate these beliefs:

One who knows it is not the equal of one who loves it, and one who loves it is not the equal of one who takes joy in it. (Book 6 pieces 20, translated by Slingerland)

A man of quality indeed was Hui! He lived in a squalid alley with a tiny bowlful of rice to eat and a ladleful of water to drink. Other men would not endure such hardships but Hui did not let his happiness be affected. A man of quality indeed was Hui! (Book 6 piece 11, translated by Dawson)

A gentleman is not motivated by the desire for a full belly or a comfortable abode. He is simply scrupulous in behavior and careful in speech, drawing near to those who possess the Way in order to be set straight by them. Surely this and nothing else is what it means to love learning. (Book 1, piece 14, translated by Slingerland)

Confucius emphasized humility. He stated, “When walking with two other people, I always find a teacher among them. I focus on those who are good and seek to emulate them, and focus on those who are bad in order to be reminded of what needs to be changed in myself” (Book 7, piece 22, translated by Slingerland).

In *Analects*, Confucius expressed his perspectives regarding teaching and learning in different books, and some of them have become precepts in China. The following pieces showed what Confucius believed about teachers’ knowledge, teachers’ passion, the teaching tasks for students, and the concrete situation for different students:

Do I possess wisdom? No, I do not. [For example, recently] a common fellow asked a question of me, and I came up completely empty. But I discussed the problem with him from beginning to the end until I finally got to the bottom of it. (Book 9, piece 8, translated by Slingerland)

I silently accumulate knowledge of things; when I study, I do not get bored; in teaching others I do not grow weary—for these things surely present me with no difficulty. (Book 7, piece 2, translated by Dawson)

Those who are better than average may talk about superior matters, but those who are worse than average may not talk superior matters. (Book 6, piece 21, translated by Dawson)

Being aware every day of what he still lacks, and after a month’s time not forgetting what he is already capable of—a person like this can be said to love learning. (Book 19, piece 5, translated by Slingerland)

To those who are not eager to learn I do not explain anything and to those who are not bursting to speak I do not reveal anything. If I raise one angle and they do not come back with the other three angles, I will not repeat myself. (Book 7, piece 8, translated by Dawson)

Confucian perspectives on learning and understanding were revealed in the following pieces. These perspectives have been cited by many comparative researchers. These researchers argued that Chinese “rote learning” originates from Confucian beliefs—memorizing with understanding.

If by keeping the old warm one can provide understanding of the new, one is fit to be a teacher. (Book 2, piece 11, translated by Dawson)

If one studies but does not think, one is caught in a trap. If one thinks but does not study, one is in peril. (Book 2, piece 15, translated by Dawson)

You, shall I teach you about understanding something? When you understand something, to recognize that you understand it; but when you do not understand something, to recognize that you do not understand it—that is understanding. (Book 2, piece 17, translated by Dawson)

Confucius attitudes to pursue knowledge are found in following pieces:

If one has heard the Way in the morning, it is all right to die in the evening. (Book 4, piece 8, translated by Dawson)

A public servant who is intent on the Way, but is ashamed of bad clothes and bad food, is not at all fit to be consulted. (Book 4, piece 8, translated by Dawson)

Give me a few more years so that I am studying at fifty, and surely I may avoid major errors. (Book 7, piece 17, translated by Dawson)

The main ideas regarding knowledge, teaching, and learning in the *Analects* can be summarized in eight aspects: value of knowledge, teachers’ role, obligation of society, goals of learning, learning process, relation of hierarchy, moral outlook, and character of students. These aspects serve as eight dimensions in the values questionnaire in my dissertation study.

Li’s (2003) study provides evidence that Confucianism is still a foundational belief regarding learning in current cultural practices. In her study, Li asked Chinese students to generate learning-related words. Many items that Chinese students generated are consistent with

Confucius' sayings in the *Analects*. For instance, the first item in Li's sorting is "Perfect oneself morally." In the *Analects*, moral issues are very important and can be found in many different chapters. In book six, Confucius answered his student's question about humaneness, "Now the humane man, wishing himself to be established, sees that others are established, and wishing himself to be successful, sees that others are successful. To be able to take one's own familiar feelings as a guide may definitely be called the method of humaneness" (translated in Dawson, 1993, p. 23). Humaneness is an ideal human quality in Confucianism; this concept is also referred to as perfect virtue, kindness, goodness, human-heartedness, and benevolence. Confucius emphasized that to become a humane man, one must become a model for others first. Other findings in Li's study revealed that current Chinese students' perceptions on learning, such as diligence, endurance of hardship, perseverance, concentration, commitment, respect, and humility for achievement, are consistent with Confucian perspectives in the *Analects*.

Confucianism is implicitly adopted in the current teaching practices in China. Two outstanding teachers, Qianxiang Zhao and Guimei Dou, introduced similar teaching strategies and beliefs in the late 1990s (Wang, 1999; Zhao, 1998, 2000). A core belief for their teaching is to help students become humane men. Zhao said, "A good Chinese teacher should keep 'humane' in mind; should not only guide students to read books, but also guide students to society and their life. To be a person is very important for students' development" (Zhao, 1998, p. 35). Both Zhao and Dou emphasized self-reflection (*Wu*), a way to seek knowledge deeply and independently. And this self-reflection must be expressed and discussed in class in order to help students learn from each other. Although they were not identified as Confucian teachings, their strategies truly reflect a Confucian belief in terms of teaching and learning. First, their strategies indicated that teaching knowledge was a means, rather than the purpose for students learning. The purpose for students' learning is to perfect themselves. This is akin to the Confucian learning goal—learning is for moral perfection. Self-reflection entails thinking actively and

diligently, and students need to experience mental struggle to get innovative ideas. As students express their ideas in public, they have to accept critiques from others. On the one hand, students can develop their humble attitude in this process. On the other hand, they can reflect their thinking again based on others' opinions. This is what Confucius said that if three people walk together, one of them must be his teacher. Zhao and Dou are recognized as two of the ten outstanding teachers nationwide, the highest honor in teacher education in China. They become models for other teachers.

Taoism is also a most influential philosophy in China's history. *Dao De Jing*, written by Laozi (604-531 B.C.) at the end of Zhou Dynasty (1027- 221 B.C.), has been perceived as classic scripture to interpret Taoist principles and doctrines. In this book, Chapter forty-two, Laozi explained that Tao gives birth to one, one gives birth to two, two gives birth to three, and three gives birth to multiple. Two indicated yin and yang, and multiple indicated everything in the world. In Taoist tradition, yin and yang are very important metaphors to explain how things work. The essential beliefs in Taoism can be interpreted as "balancing".

The important image (Figure 3.2) stands for all yin and yang principles and the relationship between Tao and yin-yang metaphors. The above image shows us two fishes putting together with opposite colors. This is the dynamic yin-yang interplay: 1) yin contains yang and yang contains yin (the two opposite eyes), 2) there is no starting point and ending point, and 3) there is no linear segment between the two fishes.



Figure 3.2: Taiji

Another equivalent image of yin and yang only used segments in ancient China. Yang corresponded to \_\_\_\_\_, and yin corresponded to \_\_\_\_\_. Both yin and yang had multiple



meanings, and they usually represented opposite things. If yin represented “physical, emotional, cerebral”; yang correspondingly represented “intelligence, energy, the spiritual, the circle” (Cooper, 1981, p.13).

The Taoist beliefs of “changing” are manifested in some Chinese ways of teaching. In the actual practices, Chinese teachers also highlight the ideas of “open-ended problem,” “different ways to solve one problem,” “generating different problems from one problem,” etc. For instance, National award teacher Sun’s three ones strategy provided an evidence of pedagogical balancing. Three ones refer to one problem with multiple solutions, multiple solutions with one problem, and multiple problems treated as one problem (Sun, 2001). The new designs of the math lesson plans (Ma, Meng, Wang, & Liu, 2002) reflected the ideas of balancing implicitly embedded in the math reform practice. These designs were mainly made by national award math teachers when they participated in the national top teacher training program in 2001. These teachers believed that math tasks in the lesson designs should contain both routine tasks and open-ended tasks in order to balance students’ thinking. As a result, one or two open-ended problems were integrated into each lesson plan. They also claimed that learning mathematics must balance the ideas of reinventing mathematics and procedurally mastering mathematics:

Balancing the ideas of reinventing and procedurally mastering mathematics is not contradictory to the current curriculum reform. Even for the students who are going to be mathematicians, we do not expect they reinvent math in each class—we do not have enough time to reinvent all. Procedurally mastering certain math content cannot be entirely disregarded. (Ma, Meng, Wang, & Liu, 2002, p. 19)

Here reinventing mathematics refers to the important concept from Realistic Mathematics Education (Freudenthal, 1973); while procedurally mastering mathematics means traditional methods that were widely applied in Chinese classroom teaching. Reinventing mathematics is consistent with the current reform math in China, and procedurally mastering mathematics is consistent with the Chinese ways to learning math before 2001—a way that both conceptual understanding and procedural proficiency are emphasized.

Unlike the influence of Confucianism, Taoist principles usually interact with other cultural elements and generate new cultural elements. For instance, “open-ended problem” does not originate in Chinese culture; “generating different problems from one problem” is also found in George Polya’s (1957) very influential heuristic methods for problem solving. Moreover, as in the West, Polya’s ideas regarding problem solving have had a big influence in China’s math education since the 1980’s. As these ideas shared the common feature “changing,” Chinese scholars and teachers easily adopted them into their theories and practices.

#### Analysis of Behaviorist and Individualist Cultural Elements

Behaviorism has a long history of impacting learning and teaching in the USA. Behaviorism holds a Reductionist position (Sawyer, 2002, p. 3). Most behaviorists treat the human brain as a black box. As a result, human intelligence only reduces to observable behaviors. From John Watson, Edward Thorndike, Ivan Pavlov, to Burrhus Skinner, behaviorists have established strict learning theories, directly applied to education for the past one-hundred years. A common research approach used by these behaviorists was animal experiments. Some principles from these animal experiments were directly used for students’ learning. In this review, I briefly introduce Thorndike’s, Pavlov’s, and Skinner’s learning theories.

Thorndike drew his law of effect by observing the cat’s behaviors in a puzzle box. The cat was hungry and tried to use various methods (e.g., pouncing, hissing, touching the loop of string) to make the door open in order to get the food outside the box. In this stimulus-response environment, the cat’s responses followed by satisfaction are more likely to occur in the same situation in the future. Thorndike (1911) explained his law of effect as follows:

Of several responses made to the same situation, those which are accompanied or followed by satisfaction to the animal will, other things being equal, be more firmly connected with the situation, so that when it recurs, they will be more likely to recur; those which are accompanied or closely followed by discomfort to the animal will, other things being equal, have their connections with that situation weakened, so that, when it recurs, they will be less likely to recur. The greater the satisfaction or discomfort, the

greater the strengthening or weakening of the bond. (cited in Zimmerman & Schunk, 2003, p. 132)

This view indicates that knowledge can be acquired incrementally. And students' behaviors in classroom can be changed through a process of reward and punishment. Thirty years after Thorndike posed his law of effect, he acknowledged that "punishment was not as effective as reward" (Zimmerman & Schunk, 2003, p. 133).

In addition to Thorndike's law of effect, Pavlov's classical conditioning and Skinner's operant conditioning were widely applied in education. A typical example to illustrate Pavlov's classical conditioning was the Pavlov dog experiment (Ormrod, 2000). If S represents stimulus, and R stands for response, the Pavlov dog experiment can be divided into three stages: 1) S (light) → R (none), 2) S (light) and S (meat) → R (salivation), and 3) S (light) → R (salivation). In the first stage, Pavlov dog did not salivate when the light flashed. In the second stage, Pavlov dog was salivating when the two stimuli—light and meat— were presented simultaneously. In the third stage, Pavlov dog was salivating when the light was presented but this time without any meat. In this case, the stimulus-response association (meat to salivation) is called unconditioned stimulus (UCS) and unconditioned response (UCR). The light in the first stage is called a neutral stimulus (p. 31). It must be along with an unconditioned stimulus in order to elicit a particular response. In the third stage, the neutral stimulus became a conditioned stimulus (CS), and its response was called a conditioned response (CR). These behaviorist concepts have directly been used in analyzing teachers' classroom behaviors. Ormrod (2000) presented a case that a teacher read a student's note to the whole class, revealing private information about that student. The classic conditioning was used by this way to conduct the case analysis:

UCS: humiliation → UCR: embarrassment in response to humiliation

CS: teacher/classroom → CR: embarrassment in response to teacher/classroom

(p. 401)

The different stimuli can elicit different feelings. Teachers were suggested to use those stimuli that can elicit feelings of happiness or relaxation. Negative comments, public humiliation, and constant frustration and failure will make students fear and dislike studying.

Skinner's operant conditioning is focused on "a class of responses" (Brumbaugh & Lawrence, 1985, p. 185). If one of the responses is followed by desirable consequences, it would tend to increase in frequency. Unlike classical conditioning, the response in operant conditioning comes first, and then is followed by a reinforcing consequence. There are three essential conditions for operant conditioning: 1) the individual must make a response, 2) a reinforcer must follow the response, and 3) the reinforcer must be presented only when the response has occurred (Ormrod, 2000, p. 405). These ideas were widely discussed and applied in education. For instance, in the second condition, researchers suggested that "the reinforcement should occur *immediately* after a desired response has occurred" (p. 405). Skinner also suggested using his teaching machines in classroom for student learning. The machines can help students learn in different pace and give immediately feedback (reinforcement). Skinner's work was also used in designing classrooms, working with mentally retarded children and adults, aiding teacher training and the like (Benjamin, 2007, p. 152).

Behaviorist perspectives about teaching and learning have a strong influence on the U.S. education. Stigler and Hiebert (1999) analyzed 1999 TIMSS video-taped lessons from the USA, Japan, and Germany. The authors argued that teaching was a cultural activity, and the typical teaching patterns drawn from the American classroom were behaviorist. Four activities characterized U.S. lessons:

- *Reviewing previous material.* The lesson begins by checking homework or engaging in a warm-up activity.
- *Demonstrating how to solve problems for the day.* After homework is checked, the teacher introduces new material, or reviews previous material, by presenting a few sample problems and demonstrating how to solve them. Often the teacher

engages the students in a step-by-step demonstration by asking short-answer questions along the way.

- *Practicing*. Seatwork is assigned, and students are asked to complete problems similar to those for which the solution method was demonstrated. Seatwork usually is done individually.
- *Correcting seatwork and assigning homework*. Near the end of the lesson, some of the seatwork problems are checked and, occasionally, some additional problems are worked out together. Homework, with more practice problems, is then assigned. (p. 81)

These activities are closely related to stimulus/response patterns—a behaviorist assumption of learning. American teachers' actions, such as checking and demonstrating, truly reflected behaviorist orientation in their teaching. A step-by-step demonstration also indicated that the teacher believed the students can imitate the teacher's work and learned knowledge incrementally.

Researchers have identified Individualism as an important Western belief both in theory and in practice. Oyserman, Coon, and Kimmelmeier (2002) argued that individualism is “a uniquely American characteristic, an integral part of their culture.” Even at the beginning, American demonstrated the individualist dispositions. As de Tocqueville described, “Such folk owe no man anything and hardly expect anything from anybody. They form the habit of thinking of themselves in isolation and imagine that their whole destiny is in their own hands” (1835/1969, p. 508, cited in Oyserman, Coon, & Kimmelmeier, 2002). Notions of *Individualism versus Collectivism* are often used in comparative studies. For instance, this pair of terms is identified as one of the four dimensions in Hofstede's (1980) cultural model. As well, some items that American students generated in Li's (2003, 2004) study reflected individualist tendency (e.g., reach personal goals, develop one's ability, being the best one can be).

Individualism/ Collectivism (I/C) construct is widely investigated and implemented. This construct is treated as either a single dimension or multiple dimensions (e.g., vertical versus horizontal Individualism/Collectivism). The majority of the research in I/C is empirical in nature.

Oyserman, Coon, and Kimmelmeier (2002) provided a thorough review on this construct and its psychological implementations.

Table 3.1: Individualism and Collectivism

	Individualism	Collectivism
Self-concept	(a) creating and maintaining a positive sense of self is a basic human endeavor (Baumeister, 1998); (b) feeling good about oneself, personal success, and having many unique or distinctive personal attitudes and opinions are valued (Oyserman & Markus, 1993; Triandis, 1995); I abstract traits (as opposed to social, situational descriptors) are central to self-definition (Fiske, Kitayama, Markus, & Nisbett, 1998).	(a) group membership is a central aspect of identity (Hofstede, 1980; Hsu, 1983; U. Kim, 1994; Markus & Kitayama, 1991); (b) valued personal traits reflect the goals of collectivism, such as sacrifice for the common good and maintaining harmonious relationships with close others (Markus & Kitayama, 1991; Oyseman, 1993; Triandis, 1995).
well-being	Open emotional expression and attainment of one's personal goals are important sources of well-being and life satisfaction (Diener & Diener, 1995; Markus & Kitayama, 1991).	(a) Life satisfaction derives from successfully carrying out social roles and obligations and avoiding failures in these domains (U. Kim, 1994; Kwan & Singelis, 1998; Markus & Kitayama, 1991); (b) Restraint in emotional expression, rather than open and direct expression of personal feelings, is likely to be valued as a means of ensuring in-group harmony.
Attribution style	Judgment, reasoning, and causal inference are generally oriented toward the person rather than the situation or social context because the decontextualized self is assumed to be a stable, causal nexus.	(a) Social context, situational constraints, and social roles figure prominently in person perception and causal reasoning (Miller, 1984; Morris & Peng, 1994); (b) Meaning is contextualized and memory is likely to contain richly embedded detail.
Relationality	Individuals need relationships and group memberships to attain self-relevant goals, but relationships are costly to maintain (Kagitcibasi, 1997; Oyseman, 1993). Relationships and group memberships are impermanent and nonintensive (Bellah et al., 1985; U. Kim, 1994; ShIder & Bourne, 1982).	(a) important group memberships are ascribed and fixed, yield as "facts of life" to which people must accommodate; (b) Boundaries between in-groups and out-groups are stable, relatively impermeable, and important; I in-group exchanges are based on equality or even generosity principles (U. Kim, 1994; Morris & Leung, 2000; Sayle, 1998; Triandis, 1995).

Oyserman, Coon, and Kimmelmeier (2002) summarized the plausible consequences of both individualism and collectivism in terms of self-concept, well being, attribution style, and

relationality (p. 5). I briefly present these terms in Table 3.1. I adopt the claims in the individualism category as basic cultural elements for our model. Some statements that can be explained by the basic cultural elements in the individualism category are identified for the study. For instance, I use the following statements in the values questionnaire to refer for the individual claims:

- The collective good is best served by individuals working in their own self interest.
- Students deserve equal attention from their teacher.
- Expressing ideas is necessary for gaining understanding.
- Confidence is the foundation for learning.
- People from impoverished background can achieve great learning if they are given special opportunities and resources to succeed.

These items are directly related to education and NCTM 2000 math standards as well. For example, the third one and the last one reflect NCTM's Equity Principle. The Equity Principle is one of the six principles in NCTM 2000 Math Standards. The Equity Principle proposes that all students must have opportunities to learn mathematics. Teachers must have high expectations and strong support for all students. Schools need to establish strong instructional programs to support all students' learning. Caring for all students in classroom indicates that teachers should pay attention to each of the students. That is, in general, teachers should use different instructional strategies for the different individual needs (e.g., to fit different learning styles, to help students expressing their ideas, and to develop students' positive attitudes toward math).

Individualism has been embedded in the actual teaching practice in American classrooms. In recent years, researchers have conducted extensive research projects to investigate how teachers implement NCTM standard-based teaching. These investigations include, but not limit to, the equity principle in classroom, communication skill enhancements, and developing

students' affects (e.g., confidence)—these are closely related to the individualist account. For instance, Cooke and Buchholz (2005) observed how a math teacher, Melissa, used informal strategies to help kids communicate in class based on the NCTM 2000 communication standards. One of the features in Melissa's informal strategies is to intently take care of each of her students, as described below:

The children were excited sometimes when they engaged in the various activities. Often, they were eager to “show” or “explain” to Melissa what they were doing. Melissa took the time to listen and respond to each child. For example, Solomon, who was using different shapes and designing figures on the overhead projector, ran over to Melissa and asked her in an excited voice to look at what he made. She walked over to the overhead and said, “Everybody look at what Solomon made! I think he did a good job. Solomon, tell me about your picture.” . . . Through her interaction with students, Melissa sent them a powerful message. Her smiles, praise, and conversations conveyed the message that they were important and they had something to say. Consequently, the children felt important and spoke with confidence. (p. 369)

Some individualist accounts are well reflected in the above paragraph. For example, Melissa likes to listen and respond to each child. Her body language and conversations convey the message that the students are important. As a result, students established confidence in expressing ideas in public.

#### Application of the New Model for Cultural Analysis in Comparative Education

In this part, I discuss three important applications of my model for cultural analysis: identifying cultural elements in the indigenous cultural context; focusing on two-level cultural analysis; and understanding the transportability of cultural practices. I will give a detailed discussion of the three aspects in the following paragraphs.

#### Identifying Cultural Elements in the Indigenous Cultural Context

In this new model, cultural elements refer to fundamental beliefs and values about teaching and learning in the indigenous cultural context. At the national level, cultural elements are manifested as precepts, sayings, and principles associated with a certain indigenous culture. At the individual level, cultural elements derive from individuals' interpretations on the certain



phenomenon regarding teaching and learning. Due to the heterogeneity of the indigenous culture, I narrow the scope of the indigenous culture when seeking its cultural elements. For example, Confucianism solely refers to the perspectives from the *Analects*. Individualist interpretations are solely based on the summary of Oyserman, Coon, and Kemmelmeier (2002) in Table 3.3. In this case, there is a clear interpretation of what accounts for the indigenous culture in each country.

Once the cultural elements are identified at the social level, one needs to examine if these elements are still active in current cultural practice. If so, these cultural elements would be used for cultural analysis.

In a broad sense, this approach can generate a set of models, rather than a single model. That is, researchers can identify different sets of cultural elements at the social level, and each set can be considered as a single model. For instance, I identify cultural elements from Confucianism, Taoism, Behaviorism, and Individualism to establish social-level cultural elements. If we try to make comparisons between Germany and Singapore, we need to use the same method to identify indigenous cultures in both countries. Other principles and theoretical underpinnings such as the emergent account of cultures still work for the model.

#### Focusing on Two-level Cultural Analysis

As mentioned above, two-level cultural analysis focuses on the cultural elements both at the societal level and the individual level and their interactions. By using this approach, I first identify cultural elements at the societal level, namely, Confucianism and Taoism in the Chinese cultural context, and Behaviorism and Individualism in the American cultural context. Then I examine their manifestations in current cultural practice. At this stage, I already have identified a set of cultural elements in each country. Two-level cultural analysis is concerned with how these social-level cultural elements interact with the individual-level cultural elements. So how to draw individual-level cultural elements, and how to connect these cultural elements with the societal-level cultural elements, becomes crucial for conducting a comparative study.

To analyze cultural elements at the individual level, both quantitative and qualitative methods are useful. I provide the following example from my dissertation study to show cultural elements can be drawn for analysis at the individual level. For a quantitative analysis, I have developed a values questionnaire based on the cultural elements identified at the societal level. For example, a 17-item questionnaire is developed for the dissertation study. My questionnaire contains ten dimensions (Table 3.2). In this values questionnaire, the items are mainly based on the social-level cultural elements I have identified in chapter two. The statements on the left side of the questionnaire reflect Confucian or Taoist perspectives, while the statements on the right side of the questionnaire stand for Individual or Behaviorism perspectives. A simple way to investigate teachers' values is to calculate the percentage of their responses in order to see a group cultural tendency. A single item in this questionnaire can be perceived as a cultural element for the qualitative analysis. For instance, in this dissertation study, I select five cultural elements from the dimensions of Teachers' Role and Learning Process as pre-determined themes for qualitative analysis.

Both Chinese and American teachers' responses in item 5, 9, and 10 are consistent with their national values. Math teachers' responses in American sample in item 11 support Eastern values. The interview and teaching episode data will help us understand more fully in items 5, 9 and 10 the concrete meanings of teaching and learning. For the item 11, I use qualitative data to analyze the cultural change in current math educational reform. This phenomenon indicates that some imported cultural elements begin to compete with the indigenous beliefs.

The above example illustrates that different sources of data can help us understand the dynamic process of cultural interactions. This is also what we call multiple realizability when we conduct analysis between the social-level and individual-level cultural elements. The qualitative data can be obtained from different sources: interview, video study, open-ended writing, actual teaching practice observation, etc. Instead of seeking all social-level cultural elements in the

individual-level practice, we believe that focusing on the fundamental aspects of the cultural elements is more important.

Table 3.2: The Relationships between the Dimensions and the Items

Dimensions	Items	Confucian and Taoist versus Individualism and Behaviorism
Value of Knowledge (VK)	1	Agency in the acquisition of knowledge (enduring pursuit versus drawing from experience)
	2	The value of knowledge ( for itself versus for its application)
	3	Knowledge and life (the essence of life versus enhancing one's life )
Structure of Knowledge (SK)	12	Relations of basic knowledge and elaborated knowledge (interchanged versus ordered)
Teachers' Role (TR)	5	Teachers' support for concept learning (providing hints toward solution versus encouraging students or reframing tasks)
	11	Pedagogical balance (variety and balance versus a single ill-chosen method)
Obligation of Society (OS)	16	Success for learners from impoverished backgrounds depends on (diligent efforts versus external support)
	8	Who merits teachers' attention (motivated students versus all students)
Goals of Learning (GL)	15	The basis for teachers' guidance (Students' different levels of understanding versus correct answers)
	4	The value of learning (enhancing one's social standing versus material success)
Learning Process (LP)	9	The ways of learning progression (through mental struggle versus through the sequenced instruction)
	10	The ways of understanding in learning (receptive versus expressive)
	14	Purposes for reviewing in the learning process (consolidate versus gaining proficiency)
Relation of Hierarchy (RH)	6	Model persons (should be emulated versus should be considered and adapted)
Moral Outlook (MO)	7	Collective good (comes from self sacrifice versus come from self-interested actions)
Attitude Relations(AR)	17	Positive and negative (coexist in all things versus selectively chosen)
Character of Students(CS)	13	The primary cultural concern (emphasizing humility versus emphasizing confidence)

## Understanding the Transportability of Cultural Practices

Understanding the transportability of cultural practices mainly helps us achieve the final goal (to learn from) in comparative education. As we introduced in chapter one, the aims of comparative study can be articulated in two phrases, “to understand” and “to learn from,” which also represent the general purposes for comparative education as agreed upon by comparative researchers (e.g., Halls, 1990; Holms, 1970; Lauwerys, 1969; Phillips & Schweisfurth, 2006). During the last two decades, researchers in comparative study have conducted extensive investigations on the first goal “to understand”. The second goal, “to learn from,” might be the final goal for comparative study.

The second goal, however, seems ambiguous in the literature of comparative study. Most comparative researchers have been interested in designing their research by asking the following questions: 1) What is the nature of? 2) What is the situation of x in the context of y? 3) How different/similar—in terms of x, is a from b in the context y? 4) Given that we can observe differences in terms of x, between a and b in the context of y, what might explain those differences/similarities? 5) What are the implications of such similarities/differences for the separate context of z (Phillips & Schweisfurth, 2006, p. 99)? The last question informs the research agenda “to learn from”. However, assimilating a good practice from the outside is extraordinarily complex. A living plant metaphor can help us understand this complexity:

We cannot wander at pleasure among the educational systems of the world, like a child strolling through a garden, and pick off a flower from one bush and some leaves from another, and then expect that if we stick what we have gathered into the soil at home, we shall have a living plant. A national system of education is a living thing, the outcome of forgotten struggles and difficulties, and ‘of battles long ago’. It has in it some of the secret working of national life. (Phillips & Schweisfurth, 2006, p. 18)

A few researchers (Thomas, 1997; Phuong, Terlouw, & Pilot, 2006) have noticed the above problems since the late 1990s. They did not believe that simply making a comparison of the differences and similarities among countries in comparative study is adequate to reach the

goal “to learn from.” These researchers warned that the national culture played a fundamental role in the adoption of a promising practice from other cultures. However, a lack of appropriate cultural models becomes an obstacle for comparative researchers to achieve the goal of “to learn from.”

The indigenous cultural model in this study can help us understand the transportability of cultural practices precisely. Based on this model, some tendencies or classroom actions cannot change in a short time if they are culturally different in both countries, and some cultural elements have been competing with each other in the current cultural practice (e.g., beliefs about humility and confidence in Chinese teachers’ responses).

### Discussion

This study provides an alternative approach to establish cultural models based on an emergentist perspective (Sawyer, 2001). This conception can accommodate the conflicting understandings in the extant models (e.g., HCH) as discussed in theoretical framework section. In this study, I apply the emergentist perspective to analyze the influence of Confucian and Taoist principles in educational research and in actual teaching practices in China. This analysis provides further understanding of culture influence on Chinese ways learning and teaching, and demonstrates how Taoist culture elements interact with other culture elements. The new cultural model presented in this study serves as a tool to analyze the influence of indigenous culture in comparative studies of teaching and learning in these two countries.

### Analysis of the NCTM 2000 and MOE 2001 Math Standards Based on Constructivist Theories

In chapter two, I have provided detailed reviews of constructivist theories and the NCTM 2000 and MOE 2001 Math Standards. Although many claim that the theoretical underpinning of the NCTM 2000 Math Standards is the constructivist theory, there is a little research revealing the relations between the constructivist theory and the NCTM 2000 Math Standards. In this

section, I first examine the theoretical rigor, namely, the forms of constructivism are presented in the NCTM 2000 and MOE 2001 Standards and their coordination. Secondly, the limitations for understanding the constructivist tenets in the two standards will be discussed.

To articulate the relations between the constructivist theory and the NCTM 2000 and MOE 2001 Math Standards, one needs to sketch the constructivist theory first. Here constructivist theory refers to two major forms and constructivist practice. The two major forms consist of radical constructivism and social constructivism. Constructivist practice refers to constructivist instructional designs such as Simon's Hypothetical Learning Trajectories and Kirshner's Crossdisciplinary Framework. Two fundamental aspects from constructivist theory will be examined: knowledge, and teachers' role. Before discussing these aspects, we recall the categorization of radical and social constructivism I reviewed in the chapter two.

### Radical and Social Constructivism in Math Education

#### Radical Constructivism in Math Education

The term radical constructivism came to math education discourse in the early 1980's. The representative of this constructivism camp is von Glasersfeld. According to Steffe and Kieren (1994), von Glasersfeld's two articles, published in *Journal of Research of Mathematics Education* in 1980 and 1981, have successfully interpreted the radical aspect of Piaget's genetic epistemology. In a later article, *Learning as a Constructive Activity*, von Glasersfeld (1987) explicitly stated both epistemological and psychological aspects of radical constructivism.

Two pairs of words can typically describe the epistemological aspects of radical constructivism: *knowing* versus *knowledge*, *fit* versus *match*. According to von Glasersfeld (1991), "Radical Constructivism is a theory of *knowing*" (p. xv), which denies objective knowledge of the world. Instead of the static status of the traditional theory of knowledge, knowing is a process through which one can deal with based on his/her experience. This basic

assumption leads to a unique way to perceive human being's communication. von Glasersfeld (1991) wrote:

If everyone had a different experiential world, they tend to argue, we could not agree on anything and, above all, we could not communicate, there is not much wrong with that argument, but the fact that we *do* agree on certain things and that we *can* communicate does not prove that what we experience has *objective reality* in itself. If two people or even a whole society of people look through distorting lenses and agree on what they see, this does not make what they see any more *real*. (p. xvi)

The above arguments lead to a conclusion that one can construct viable knowledge. von Glasersfeld (1987) claimed that “it is in this context [a teacher models children's concept] that the epistemological principle of *fit*, rather than *match*, is of crucial importance” (p. 13). This epistemological principle of fit also indicates there is no way to access one's reality—the only way we know others is to make hypotheses about the reality. These basic tenets resulted in a new research methodology called the Constructivist Teaching Experiment in math education.

In terms of psychological aspects of radical constructivism, von Glasersfeld highlighted motivation in his 1987 article, and the terms from Piagetian tradition such as assimilation, accommodation, perturbation. The motivation can be from the inside of the organism's own system in order to achieve a satisfactory organization. For example, when children play puzzles or wooden blocks, the rewards spring from their achievement rather than from the outside. This perspective has been adopted by Simon and Tzur (2004) to explain his activity-effect relationships that served as a mechanism in math concept development (p. 92). Although researchers who held radical constructivist beliefs also adopted Piaget's notions, such as assimilation and accommodation, to explain the process of cognitive construction, they entirely abandoned Piaget's stage theory which was largely criticized by academic community.

To my understanding, the relationships between Piaget and von Glasersfeld constructivism theory can be articulated as follows:

- Both Piaget and von Glasersfeld held the view that knowledge is constructed based on learners' past experience.

- Piaget did not clearly state the notion of constructivism. His main interest was to develop his stage theory. As a psychologist, Piaget took the concept of *adaptation* seriously in his study.
- von Glasersfeld adopted Piaget micro-genetic epistemology to develop his radical constructivism theory. As a philosopher and psychologist, von Glasersfeld took the concept of *self-organization* and the viability principle seriously in his study.
- von Glasersfeld's radical constructivism has integrated in some math education programs developed by Steffe, Cobb, Yackel, Simon, and others. In contrast, the applications of Piaget's theory were not very successful in math education, as Steffe and Kieren (1994) reviewed.

### Social Constructivism in Math Education

Social constructivism, as Ernest pointed out, entered mathematics education field as a philosophy at the 1980's. In social psychology, Harre, Gergen, Shotter, Coulter, Secord discussed the social construction of the self, personal identity, gender and the like in the 1970's. Researchers (e.g., Weinberg & Gavelek, 1987; Bishop, 1985; Ernest, 1990, 1991a, 1991b, 1994, 1998; Bauersfeld, 1992; Bartolini-Bussi, 1991) have conducted studies with the use of social constructivism directly or indirectly. Ernest (1994) clarified the two kinds of social constructivism: social constructivism with a Piagetian theory of mind and social constructivism with a Vygotskian theory of mind. And in 1998, Ernest offered a deeper analysis of social constructivism as a philosophy of mathematics.

According to Ernest (1998), Wittgenstein's and Lakatos' contributions have been under-recognized. Based on Wittgenstein's language game and Lakatos' logic of mathematics discovery (LMD), Ernest identified "the social construction of subjective and objective knowledge of mathematics" (p. 241). He claimed,

At the center of social constructivism lies an elaborated theory of both individual or subjective knowledge and social or objective knowledge—equally weighted (although in traditional epistemology the latter is prioritized)—and the dialectical relation between them. There is, first of all, a powerful structural analogy between subjective and objective knowledge of mathematics through the role of conversation. For the two types of voice in conversation are those of the knowledge constructor (proponent) and critic, types that figure in the construction and warranting of both personal and public knowledge of



mathematics. Second, these types of knowledge are dialectically interrelated and implicated in each other's creation and warranting. (p. 241)

Ernest pointed to a broader sense of math knowledge. He counted Popper's three-world knowledge— physical world, conscious experiences, and contents of books and libraries—as objective knowledge. Therefore, math theories, axioms, problems, conjectures, and proofs are perceived as objective knowledge. In his classification, know-how, propositional knowledge, tacit, and explicit knowledge also belong to math knowledge.

Aside from the concern of the philosophical aspect of social constructivism, we still need to introduce Vygotsky's theory. Vygotsky was influenced by emergence theories that were prominent during the 1930s. Sawyer argued that Vygotsky was a sociological holist, "because he did not attempt to explain social phenomena themselves in terms of how they emerged from individuals and interactions" (Sawyer, 2002, p. 15). This point was reflected in Vygotsky's critique on the element analysis in psychological research. Vygotsky (1934, translated by Hanfmann & Vakar, 1962) argued element analysis can be compared to the chemical analysis of water into hydrogen and oxygen. The whole properties of water cannot be found by analyzing the elements of hydrogen and oxygen.

Instead of element analysis in psychology, Vygotsky developed a new method called *analysis into unites*. He wrote,

By *unite* we mean a product of analysis which, unlike elements, retains all the basic properties of the whole and which cannot be further divided without losing them. Not the chemical composition of water but its molecules and their behavior is the key to the understanding of the properties of water. The true unit of biological analysis is the living cell, possessing the basic properties of the living organism. (Vygotsky, 1934, translated by Hanfmann & Vakar, 1962)

Vygotsky's perspective of unite analysis is consistent with the key point held by the social holists that macro-social phenomena cannot be redefined as individual behavior, a non-reductionist perspective.

Vygotsky's perspective on development can be briefly summarized as "Every function in the cultural development of the child appears on the stage twice, on two planes. First, on the social plane, and then on the psychological; first, between people, and then, inside the child" (Vygotsky, 1987; cited in Vasily V. Davydov, 1995, p. 16). Vygotsky put the social plane as primary, in opposition to Piagetian radical constructivist perspectives on the child development. Unlike the radical constructivist von Glasersfeld, who claimed the construction process was inherently pleasurable for humans, Vygotsky did not believe "that learning related to the zone of proximal development is always enjoyable" (Chaiklin, 2003, p. 43).

Vygotsky's Zone of Proximal Development (ZPD) has been applied in many projects (e.g., Tharp & Gallimore, 1988; Murata & Fuson, 2006, Steele, 2001) in math education, although it is not a central concept of his theory of child development. The following quotation is well documented as the definition of ZPD:

The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. (Vygotsky, 1978, p.86. cited in Chaiklin, 2003, p. 40)

According to Chaiklin (2003), the common conceptions of ZPD can be interpreted as three aspects: generality assumption, assistance assumption, and potential assumption. The first one hypothesizes a student can perform a greater number of tasks in a collaborative situation than he/she can independently. The second one focuses on the positive influences of competent people in the students' learning situation. The third one inspires the expectation that the child can be accelerated within the potential space. Chaiklin (2003) further summarized that the main features of the analysis of ZPD are the five aspects: a) whole child, b) internal structure, c) development as a qualitative change in the structural functions, d) brought about by the child's actions in the social situation of development, and e) each age period has a leading activity to develop new functions (p. 50).

## Analysis of the Relationships between Constructivist Theory and the NCTM 2000 and MOE 2001 Math Standards: Knowledge and Teachers' Role

Two theoretical aspects of constructivist theory will be examined in the NCTM 2000 and MOE 2001 Math Standards: 1) knowledge and 2) teachers' role.

### Knowledge

As I reviewed before, a radical claim of knowledge is of knowledge as *knowing*, and different people know the world differently. Of the six principles, NCTM 2000 Math Standards set the Equity Principle as its first. The Equity Principle challenges the view that only able students learn mathematics. This perspective implies that math knowledge is not the static logical objects that are outside of learners' minds; Different students can learn mathematics differently.

The differences in NCTM 2000 Math Standards can be understood in two aspects. The first is that NCTM 2000 Math Standards acknowledge individual differences in terms of students' prior experiences, personal characteristics, and intelligence. The second is that NCTM 2000 Math Standards claim that mathematical content should be presented in different ways. For instance, NCTM 2000 Math Standards advocates multiple methods to solving the same problem, at the same time stressing informal representations of math knowledge. Students should take advantage of their prior experience and their own ways to think about and solve a problem in their learning. An example in NCTM Math Standards showed this:

Consider the following problem that might be used by a teacher who wants her students to think about various ways to use ratios and proportions:

A baseball team won 48 of its first 80 games. How many of its next 50 games must the team win in order to maintain the ratio of wins to losses?

Students can solve this problem in many ways. One student might express the ratio of wins in the first 80 games as  $48/80$  and note that the ratio is a little more than one-half; that is, the team wins a little more than half the time. She might then estimate that in the next 50 games the team should win about 28 games. She could compare the resulting ratio  $28/50$  to the given ratio of  $48/80$  and adjust her estimate until the two ratios are equivalent. Another student might look at the ratio of wins to losses, . . . , Yet another student might use a proportion,  $48/80=x/50$ , to find the solution. . . . (NCTM 2000, p. 257)

To some extent, these ways of envisioning the students' possible thinking together with students' informal representations agree with what radical constructivists have called "Mathematics of Students." This is different from "Students' Mathematics" (Steffe & Thompson, 2000). In Steffe and Thompson's study, Students' Mathematics is the students' math reality, and Mathematics of Students is what researchers or teachers assume about the students' reality. Researchers and teachers interpret mathematics of students based on their own experience and the information from the literature. These concepts are based on the unknowable nature of knowledge in von Glasersfeld's claim:

We come to see knowledge and competence as products of the individual's conceptual organization of the individual's experience, the teacher's role will no longer be to dispense "truth" but rather help and guide the student in the conceptual organization of certain areas of experience. Two things are required for the teacher to do this: on the one hand, an adequate idea of where the student is and, on the other, an adequate idea of the destination. Neither is accessible to direct observation. What the students says and does can be interpreted in terms of a hypothetical model---and this is one area of educational research that very good teacher since Socrates has done intuitively. (von Glasersfeld, 1987, p. 16)

I do not claim that NCTM 2000 Math Standards adopt radical constructivist tenets explicitly, rather, they do not explicitly state what accounts for math knowledge. What the NCTM 2000 Math Standards assume about math knowledge combines radical constructivist and social constructivist perspectives. As I reviewed in the literature, informational representations together with students' own ways to solve problems can be perceived as subjective knowledge from the social constructivist perspective (Ernest, 1998). Ernest advocates that math theories, axioms, problems, conjectures, and proofs are all perceived as objective knowledge. And the knowledge, both subjective and objective, is constructed through conversation. For example, students' knowledge can be acquired through the Zone of Proximal Development (ZPD). Some perspectives in NCTM 2000 Math Standards such as formal representations, mathematical reasoning, and proofs can be perceived as objective knowledge. In terms of the relationship between subjective and objective knowledge, NCTM 2000 Math Standards hold that students

should acquire this knowledge gradually through their K-12 studying (for example, p. 57; p. 59; p. 71). This claim is not shared by social constructivist theory. However, both social constructivist theorists and NCTM 2000 Math Standards advocate that knowledge should be constructed through communication (conversation). In NCTM 2000 Math Standards, communication contains four aspects:

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely. (p. 60)

The first aspect claims communication benefits students in the following ways: learning new math concepts, identifying misconceptions, sharing responsibility with teachers, and consolidating students' thinking through writing. This emphasizes constructing subjective knowledge from a social constructivist perspective.

The second aspect suggests that teachers should build a community “in which students will feel free to express their ideas” (p. 61). Students in different grades have different requirements in order to think coherently and clearly. Students in the lower grades need more help from their teachers to share ideas than students in other grades. Students in grades 3-5 can participate in whole class discussions. Teachers should create communication-rich environments for middle school students, although middle school students do not like to stand out in group discussions. This aspect shares in part ideas from Vygotsky's Zone of Proximal Development (ZPD). That is, Students in lower grades should receive more help from their teachers in order to reach a higher level of achievement that cannot be achieved by self-learning. Peer assistance might be of benefit from third through fifth graders. And students in middle school might be benefit from both teachers' and their peers' help.

The fourth aspect returns again to the point of view in ZPD. The fourth aspect in communication standards suggests that teachers should help lower grades students discriminate the same words with different meanings as they are used in mathematical expression. Teachers also should avoid “a premature rush to impose formal mathematical language” (p. 63). Clearly NCTM 2000 Math Standards assumes that teachers possess more knowledge than students’—a view of knowledge that Vygotsky held but to which radical constructivists object. This point also can be found in the following statement: “The teaching principle makes the case that students must have opportunities to learn important mathematics under the guidance of competent and committed teachers” (NCTM, 2000, p. 7).

To summarize, although NCTM 2000 Math Standards do not explicitly claim what accounts for math knowledge, they have implicitly adopted the view of knowledge from both radical and social constructivist perspectives. In terms of knowledge construction, NCTM 2000 Math Standards tend toward a social constructivist position without discarding the radical constructivist point of view.

Chinese MOE 2001 Math Standards takes a similar view regarding math knowledge compared to NCTM 2000 Math Standards. For example, both the NCTM 2000 and MOE 2001 Math Standards claim teaching for all students, and different students should exhibit different development in mathematics. Both the NCTM 2000 and MOE 2001 Math Standards emphasize that learning tasks should connect with real life and students’ past experiences. Moreover, MOE 2001 Math Standards heavily emphasizes “learning process”; this assumes that students should learn by participating in problem solving activities. This implies math knowledge should be built on students’ prior experience and constructed by the students’ participation. Therefore, it seems that MOE 2001 Math Standards is closer to a social constructivist view of knowledge and its construction.

## Teachers' Role

Both NCTM 2000 and MOE 2001 Math Standards advocate that a teacher should serve as a facilitator, a co-participant, and an organizer in the classroom. In the Teaching Principle, NCTM appears to claim that teachers should be concerned with three elements to conduct an effective mathematics teaching: students' prior knowledge, a balanced use of pedagogies, and a focus on students' dispositions (NCTM, 2000, p. 18). The first two elements of teachers' role (connecting students' prior knowledge and a balanced use of pedagogies) imply both radical and social constructivist perspectives. Likewise, the third element of the teachers' role (fostering students' dispositions) implies a social constructivist perspective.

The first two elements imply both radical and social constructivist perspectives. The first element, connecting students' prior knowledge, has two meanings in NCTM 2000 Math Standards: 1) students learn by connecting new ideas to prior knowledge; and 2) teachers should select tasks that build on students' prior knowledge. The first meaning states the learning principle, while the second meaning states the teachers' role in selecting learning tasks. In fact, neither radical nor social constructivists deny the important role of prior knowledge in students' learning. Radical constructivists (e.g., von Glasersfeld, Steffe) advocate the point that students' learning depends on their own past experience. To some extent, students' past experience is partially equivalent to their prior knowledge. Social constructivists believe that students can make progress under adult guidance or in collaboration with more capable peers (e.g., Vygotsky's ZPD). This implies that adult guidance or capable peers possess much more prior knowledge than the learners. From the perspective of radical constructivists', the teachers' role as a facilitator and co-participant means that teachers should focus on how to engage students in goal-directed mathematical activity (Steffe, 1991). In this type of activity, teachers should make judgments on Mathematics of Student that reflect students' prior knowing, and what are the conceptual obstacles when students engage in the new tasks. Students learn and make progress

by themselves. By contrast, social constructivists believe that a student can perform a greater number of tasks in a collaborative situation than independently. In other words, students can make progress through social interactions. It appears that teachers play a more important role in ZPD learning than in a radical constructivist learning environment.

The second element of teachers' role in NCTM 2000 interpretation (a balanced use of pedagogies) also implies both radical and social constructivist perspectives. NCTM describes such a balanced use of pedagogies:

One of the complexities of mathematics teaching is that it must balance purposeful, planned classroom lessons with the ongoing decision making that inevitably occurs as teachers and students encounter unanticipated discoveries or difficulties that lead them into uncharted territory. (p. 18)

The above statement indicates that NCTM does not advocate developing a fixed lesson plan. The teachers' role is to develop a tentative lesson plan and to make changes in the classroom. This ongoing decision-making is very important for radical constructivist teaching. Since radical constructivists believe that students' realities are unknowable, teachers should instantly assess Mathematics of Students (Steffe & Thompson, 2000). Social constructivists (e.g., Lesh & Yoon, 2004) also advocate ongoing-decision making in classroom.

In short, NCTM describes the three elements regarding teachers' role in the Teaching Principle. The first two elements coordinate radical and social constructivist theories. The third element, fostering students' dispositions, is consistent with a social constructivist stance. The overall interpretations of the teachers' role are more consistent with social constructivism.

Aside from the above three elements of teachers' role in the Teaching Principle, NCTM also states teachers' role in the Process Standards separately. For instance, in grades 6-8 problem solving standards, NCTM mainly specifies teachers' role as follows:

- To help students develop a problem-solving orientation,
- To help build students' problem-analysis skills by including tasks that have extraneous information or insufficient information,



- To motivate students by encouraging communication and collaboration and by urging students to seek compete solutions to challenging problems,
  - To counteract negative dispositions (e.g., all problems can be solved quickly and directly; there is only one right way to solve a math problem),
  - To help students become reflective problem solvers by frequently and openly discussing with them the critical aspects of the problem-solving process.
- ... (pp. 258-259)

NCTM's interpretations above regarding teachers' role in problem solving further verify a social constructivist orientation. Three of them concern nurturing students' dispositions.

Chinese MOE 2001 Math Standards lays out teachers' role from a social constructivist stance. This suggests that math teaching is activity-based teaching in nature. Teachers should take a role equal to the students in class. They should become co-participants, facilitators, and organizers. Teachers should encourage students to explore knowledge both independently and collaboratively.

The above analysis indicates that the NCTM 2000 and MOE 2001 Math Standards reveal both radical and social constructivist perspectives regarding knowledge and teachers' role. A fundamental claim in NCTM 2000 Math Standards is that "all students should learn important mathematical concepts and processes with understanding" (p. ix). This claim accords with constructivist beliefs. If NCTM takes constructivism as the philosophical and epistemological underpinning for their 2000 Math Standards, there are several limitations regarding their claims regarding the teachers' role.

The first limitation is that NCTM does not articulate a coherent constructivist theory in the heterogonous constructivist discourse to base their arguments on teachers' role. It would be clearer if NCTM explained openly what kind of constructivist theory they adopted to describe the teachers' role.

The second limitation is that NCTM does not articulate the relationships between teachers' role and teaching goals. If NCTM only takes teaching for understanding as a general teaching goal, the teachers' role of nurturing students' dispositions in NCTM's Teaching Principle is not appropriate for that goal. In fact teachers cannot help students understand a disposition; rather, teachers can help students develop a disposition over a long time period. In addition, NCTM advocates the statement that ongoing decision-making in classrooms is part of the teachers' role. It seems that NCTM should further state how this ongoing decision making might affect the teaching goals. NCTM repeatedly claims that teachers should take their role in task-selection without discussing teaching goals. However, the task may be used quite differently if the teaching goals are different. For instance, NCTM advocates math problems that can be solved in different ways. If teachers select this teaching task and set up a goal of nurturing students' disposition regarding divergent thinking, teachers might strongly encourage students to find different ways to solve the problem. If teachers set a goal of helping students understand concepts more fully, teachers might focus on the differences between the methods, rather than the diversity of the methods, *per se*.

The third limitation is that NCTM and MOE do not point out how to prepare and select learning tasks that can meet the needs of all students in classroom instruction. The Equity Principle in NCTM 2000 Math Standards clearly states that "equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (p. 12). The similar claim also can be found in MOE 2001 Math Standards. However, selecting effective tasks is not adequately discussed in NCTM and MOE Math Standards. It seems that hands-on, manipulatives, and real-world tasks are strongly recommended by both Standards. These tasks seemingly meet social constructivist perspectives because it is easier to conduct an activity-based lesson by using these tasks. In fact, individual differences are a barrier for accomplishing real

learning for all students. In learning literature for gifted students, researchers (Colangelo, Assouline, & Gross, 2004) found that the learning pace for gifted students is more than eight times as fast as normal students. By contrast, some low-performed students need special aids in learning math. NCTM and MOE do not propose a concrete way to resolve this dilemma.

To summarize, NCTM and MOE have addressed math knowledge and teachers' role through constructivist lenses. Both radical and social constructivist perspectives are reflected in the two math standards. Constructivist perspectives regarding knowledge result in new ways to perceive teachers' role, activity design, learning process, and the like in both standards. These perspectives are quite different from Behaviorist teaching and learning. Although NCTM and MOE take constructivism as their theoretical underpinning for their math standards, they do not explicitly articulate how to adopt heterogeneous constructivist perspectives to make coherent arguments with regard to teachers' role. Teachers' role is related to teaching goals; both NCTM and MOE do not discuss this adequately. Another limitation discussed before is if NCTM and MOE make selecting effective learning tasks one of the teachers' roles. How does selecting these tasks meet the Equity Principle?

One point that must be articulated is that I do not claim that all limitations are caused by the disconnection between constructivist theories and NCTM/MOE interpretations. In fact, constructivist theories cannot provide adequate support for the issues NCTM and MOE addressed in the two standards. For instance, before 2000 researchers did not discuss well the constructivist-based instructional designs until 2004 when Simon and Tzur published their elaboration of Hypothetical Learning Trajectory. Skills and dispositions were not articulated well before 1998 (see Sfard, 1998; Kirshner, 2002). Recent theorizations (e.g., Sfard's two metaphors' learning, Kirshner's Crossdisciplinary Framework) may provide new angles for improving and revising NCTM and MOE Math Standards.

## CHAPTER FOUR: METHODOLOGY AND PROCEDURES

### Design of the Study

The research was designed as a comparative case study. The QUAL-quant mixed methodology was used for this study, which means the research is explanatory in nature, and the qualitative data is the main part of the analysis (Creswell, 2005, p. 520). Thirty middle school teachers in each country were selected in this study, teachers who used the reform math approach in their classes.

Purposeful sampling (Creswell, 2005, p. 204) was used in this study. In China, thirty middle school math teachers were selected from the Changchun area including Changchun city and its five counties. American counterparts were selected from the Baton Rouge area including East Baton Rouge Parish and West Baton Rouge Parish.

A contact person assists me in finding subjects and collecting data in each country. In China, the contact person is Zhang Anli, who knows the middle schools in the Changchun area very well. She is an educational researcher in the Educational Institute of Jilin Province with more than twenty years of experience with middle school math teaching. She was able to contact local math educators in Changchun area. They met together to decide the Chinese sample based on my requirements: subjects attempting to implement reform ideas in their teaching. They first selected the middle schools, and then the schools' principals helped get the subjects. In the USA, Ms Yoon, a coordinator in teacher education center at Louisiana State University, helped me find the middle school reform teachers. Each participant was paid \$20 for filling out the questionnaire, while each of the twelve interviewees was paid for \$30 for a one-hour interview. I thank SPSSI and College of Education at LSU for providing funds to my study.

## Instrument

The Constructivism-Culture and Actual Teaching Practice Survey (CCATPS) was developed and used in this research. In order to get rich and in-depth information for data analysis, the CCATPS contains four parts. Part I is a reform-orientation questionnaire; part II is a teaching-style questionnaire; part III is a values questionnaire; and part IV is the free writing of teachers about their actual teaching practice.

Table 4.1: Dimensions and Items in the Reform Orientation Questionnaire

Dimensions	Items	Items
D1: Program scope	4	I tend to integrate multiple strands of mathematics with a single unit
D2: Student tasks	1	I like to use math problems that can be solved in many different ways
	2	I regularly have my students work through real-life math problems that are of interest to them
D3: Discovery	14	I don't necessarily answer students' math questions but rather let them puzzle things out for themselves
D4: Teacher's role	5	I often learn from my students during math time because my students come up with ingenious ways of solving problems that I have never thought of
	17	I teach students how to explain their mathematical ideas
D5: Manipulatives and tools	10	I encourage students to use manipulatives to explain their mathematical ideas to other students
	18	Using computers to solve math problems distracts students from learning basic math skills
	19	If students use calculators they won't master the basic math skills they need to know
D6: Student-student interaction	3	When two students solve the same math problem correctly using two different strategies, I have them share the steps they went through with each other
	6	It is not very productive for students to work together during math time
	9	In my classes, students learn math best when they can work together to discover mathematics ideas
D7: Student assessment	8	I integrate math assessment into most math activities
	12	Creating rubrics for math is a worthwhile assessment strategy
D8: Teacher's conceptions of math as a disciplinary	15	A lot of things in math must simply be accepted as true and remembered
D9: Student confident	20	You have to study math for a long time before you see how useful it is

Part I is an existing questionnaire in the literature developed by Ross, McDougall, Hogaboam-Gray, and LeSage (2003) with a high reliability ( $\alpha = .81$ ). This questionnaire was designed for K-8 school teachers' self-reports on their reform teachings. The higher the accumulated score a teacher obtains, the more reform orientation the teacher tends to be. Based on NCTM standards and principles, the reform ideas were divided into nine dimensions in this survey (see Table 4.1).

The purpose of collecting Part I data was to confirm whether subjects are reform teachers. Although all subjects selected have been identified or self-identified as reform teachers, this needed to be confirmed. Each subject received a score by accumulating his/her answers from the self-report survey. I set the cutoff score as 60, the mid-point of the total score range of 20 to 100. A score of 60 or greater was taken to indicate that a teacher is reform-oriented. New subjects would have been invited to join in this study if some subjects were identified as traditional teachers.

Part II is a teaching style questionnaire developed for this study. The theoretical foundations for Part II are Radical Constructivism and Social Constructivism. Radical Constructivism adopted here is mainly based on von Glasersfeld's interpretation, in contrast to Vygotskian oriented theories of Social Constructivism. In addition, some perspectives are adopted that have extended our understanding of constructivist teaching and learning.

Part II data can help us to understand the relations between theoretical perspectives of constructivism and NCTM 2000 and MOE 2001 Math Standards. This understanding was not achievable by administering extant questionnaires that are only concerned with a reform-tradition dimension. I will discuss this point later.

In part II, the items on the left side are informed by Radical Constructivism, whereas the items on the right side are informed by Social Constructivism. Part II contains three dimensions: Learning Process, Teachers' Role, and Learning Goal. These dimensions are the fundamental

concerns for both Radical and Social Constructivism. In short, the relationships between the dimensions and the items are demonstrated in Table 4.2.

Table 4.2: Dimensions and Items in the Teaching Style Questionnaire

Dimensions	Items	Radical Constructivism versus Social Constructivism
Learning Process (LP)	4	The effectiveness of lecture (useful for students understanding versus not useful for students' understanding)
	8	Ontology of conceptual learning ( individual reflective practice versus acts of communication)
	10	The origin of motivation (the innate drive to understand versus social participation)
Teachers' Role (TR)	1	Teachers' and students' roles in knowledge production (co-producing in learning activity versus guidance)
	3	Facilitating students' learning (focus on the content of studies and students' thinking versus organizing processes of students' discussion)
	5	The source of task (from teachers versus from students)
	6	The design of tasks (procedural instruction versus emergency in the process of instruction)
	7	Open-ended and manipulative tasks (essential versus supplementary)
Learning Goal (LG)	2	The goal of learning (content understanding versus developing cognitive dispositions)
	9	The purpose of using multiple solutions for the same problem (deeply understand concepts versus flexible thinking)

Part III is a seventeen-item cultural questionnaire developed for this study. The theoretical foundation of Part III is the indigenous culture model of teaching and learning in Chapter four Section two. This culture model perceives Confucian and Taoist culture as the fundamental roots in terms of teaching and learning in China, in contrast with Behaviorism and Individualism in the USA. In Part III, the statements on the left side reflect Confucian or Taoist perspectives, while the statements on the right side stand for Individual or Behaviorism perspectives. Part III has ten dimensions: Value of Knowledge, Structure of Knowledge, Teachers' Role, Obligation of Society, Goals of Learning, Learning Process, Relation of

Hierarchy, Moral Outlook, Attitude Relations, and Character of Students. The relationships between dimensions and items are illustrated in Table 4.3.

Table 4.3: Dimensions and Items in the Values Questionnaire

Dimensions	Items	Confucian and Taoist versus Individualism and Behaviorism
Value of Knowledge (VK)	1	Agency in the acquisition of knowledge (enduring pursuit versus drawing from experience)
	2	The value of knowledge ( for itself versus for its application)
	3	Knowledge and life (the essence of life versus enhancing one's life )
Structure of Knowledge (SK)	12	Relations of basic knowledge and elaborated knowledge ( interchanged versus ordered)
Teachers' Role (TR)	5	Teachers' support for concept learning (providing hints toward solution versus encouraging students or reframing tasks)
	11	Pedagogical balance (variety and balance versus a single well-chosen method)
Obligation of Society (OS)	16	Success for learners from impoverished backgrounds depends on (diligent efforts versus external support)
	8	Who merits teachers' attention (motivated students versus all students)
Goals of Learning (GL)	15	The basis for teachers' guidance (Students' different levels of understanding versus correct answers)
	4	The value of learning (enhancing one's social standing versus material success)
Learning Process (LP)	9	The ways of learning progression (through mental struggle versus through the sequenced instruction)
	10	The ways of understanding in learning (receptive versus expressive)
	14	Purposes for reviewing in the learning process (consolidate versus gaining proficiency)
Relation of Hierarchy (RH)	6	Model persons (should be emulated versus should be considered and adapted)
Moral Outlook (MO)	7	Collective good (comes from self sacrifice versus come from self-interested actions)
Attitude Relations (AR)	17	Positive and negative (coexist in all things versus selectively chosen)
Character of Students (CS)	13	The primary cultural concern (emphasizing humility versus emphasizing confidence)



The main purpose of collecting Part II and Part III data was to calculate the percentage of the subjects' responses for selected items. This simple statistical information was used to contrast with or compare to qualitative data from Part IV and interviews. Likewise, Part II and Part III helped extend the understanding of the subjects' beliefs and values in order to write meaningful interview protocols. For example, the tendency for responding to Radical Constructivist teaching might be different between teachers in China and the U.S. This tendency might be highly related to certain cultural values demonstrated in Part III. This kind of information helped us understand the influence of indigenous culture in each country and transportability of teaching practices between the two countries.

Part II data also helped us to understand the relations between theoretical perspectives of constructivism and NCTM 2000 and MOE 2001 Math Standards. This understanding was not achievable by administering extant questionnaires that are only concerned with a reform-tradition dimension. For instance, a questionnaire dealing with reform-tradition teaching and learning only identifies that advocating "using multiple methods for solving the same problem" is a reform teacher's claim. However, I am also interested in the teachers' understandings of the purpose and tendency of using multiple methods for solving the same problem. That is, the teachers may tend to favor either a Radical Constructivist belief that "The main purpose of using multiple solutions for the same problem is to help students deeply understand concepts" or a Social Constructivist belief that "The main purpose of using multiple solutions for the same problem is to help students develop their flexible thinking habit" or both.

Part IV consists of qualitative data scenarios that the subjects were asked to come up with. The subjects were asked to recall an actual teaching scenario in their careers (10-20 minutes) in which students really seemed to be having strong learning experiences. The subjects are expected to explain their roles, the key points in their design, student-teacher interactions, unexpected events, and the like in details.

The CCATPS survey was written in English first, and then translated into Chinese. Chinese teachers were given CCATPS in the Chinese version. An assistant professor who knows both Chinese and English helped with the back-translation. That is, she translated the Chinese version of the survey into English in order to check if my translations were correct.

A pilot study had been conducted before data collection. Three math teachers were selected in each country to complete the survey. A few items were revised based on the pilot study.

## Interviews

Six subjects in each country were selected for a one-hour interview that follows up and extends the responses in the survey. The following method was used for choosing the interviewees. When subjects returned the surveys, their basic information, such as their names, contact information, and years of teaching, was included in the first page. Ten teachers in each country were selected based on their teaching experiences, schools, and genders. After the first selection, I emailed these teachers to invite them to participate in an interview. Six teachers in each country were finally selected according to their agreement to participate.

Telephone interviews were used with Chinese subjects, while face-to-face interviews were used with the USA subjects. All interview data were audio-taped. Although the data collection methods (telephone and face-to-face interview) were different for the two countries, this difference would help compensate for the fact that the Chinese interviews were conducted in my native language.

A semi-structured interview protocol was designed for interviewees in both countries. The following seven questions were prepared for the interviews. The last two were based on teachers' response in their questionnaires.

- What is your perspective on the math reform teaching movement? (and other general information)

- Please tell me your personal experience in classroom teaching. Do you like to use student-centered pedagogy? (Chinese subjects will be asked to describe teacher-centered pedagogy used in their classes)
- NCTM (or MOE) claims that teaching math is for understanding math concepts. What is your perspective with regard to understand math concepts and mastery of math skills?
- Please describe your actions in the classroom for keeping students motivated who may have very different cultural backgrounds.
- Tell me something that you are struggling with in your efforts at reform math teaching.
- Two concrete questions related to indigenous culture will be drawn from their actual teaching practice questionnaire.

The interview helped us collect data on teachers' beliefs and values with regard to teaching and learning. The interviewees' interpretations revealed their struggles when implementing reform lessons. The interview data demonstrated the relations between their indigenous beliefs and constructivist beliefs. The transportability of the practices across country boundaries was analyzed by looking at these relations.

The first question was designed to get the general information and to create a good rapport for the conversation. The second question was to test interviewees' preference of classroom teaching formats. Both Chinese 2001 math standards and American NCTM 2000 Math Standards tend to advocate student-centered teaching format. Chinese teachers are accustomed to teacher-centered teaching, which has been used for one-hundred years. From a cultural perspective, teaching for understanding with teacher-centered pedagogy is deeply rooted in Confucian and Taoist culture. In contrast, American education is influenced by heterogeneous psychologies and philosophies, among which I identify behaviorist and individualist beliefs and

values as most influential in teaching and learning. No matter what kind of teaching format is advocated for the class, the influence of the indigenous culture would make teachers' actions different. A series of questions regarding teaching formats was introduced in the interview conversations.

The third question invites teachers' perspectives with regard to understanding math concepts and mastery of math skills. Both math standards in China and the USA demonstrate a feature that teaching for understanding is heavily emphasized. Taoist beliefs highlight a balanced perspective; while Confucianism contends that proficiency and memorizing knowledge facilitate deeper understanding. Influenced by these traditional beliefs, Chinese math educators had developed their own teaching strategies for both conceptual understanding and skill acquisition in the past. How Chinese teachers deal with reform-oriented teaching becomes an interesting topic for investigation. In the USA, behaviorism has a long term influence on teaching, and behaviorist teaching primarily benefits skill acquisitions. Moreover, an individualist tradition does not support classroom authority. How American teachers strive to get rid of behaviorist influence toward to teach for understanding becomes an important issue in data collection. A series of questions regarding skills and concepts would come out in the interview conversation. In addition, the second and the third questions are key aspects in the discussion part of my dissertation.

The fourth question is highly related to Confucian and individualist beliefs. The fifth question tries to draw something that the interviewees have struggled with for a long time. The sixth and seventh questions were later developed based on the survey data.

### Data Analysis

Three types of data were analyzed in this study: 1) document data from constructivist theories, NCTM 2000 and MOE 2001 Math Standards, and current cultural related perspectives in terms of teaching and learning, 2) data from Constructivism-Culture and Actual Teaching

Practice Survey (CCATPS) for all subjects, and 3) interview data from both countries. The document data led to a new cultural model and the theoretical evaluation of NCTM 2000 and MOE 2001 Math Standards from a constructivist lens. The empirical results were drawn from the survey and interviews. The limitations and theoretical rigor of NCTM 2000 and MOE 2001 Math Standards were analyzed based on constructivist theories. The new cultural model served as a framework for the analysis of the empirical data.

The document data were analyzed theoretically. Constructivist theories were reviewed based on von Glasersfeld's 1991 and Ernest's 1998 interpretations of radical constructivism and social constructivism, respectively. More recent studies in constructivist teaching and learning are synthesized and analyzed in the review. For instance, hypothetical learning trajectories (e.g., Simon, 1995; Simon & Tzur, 2004; Steffe, 2004) and teacher-centered constructivist pedagogy (Kirshner, 2004) were introduced and analyzed. These new developments and elaborations of constructivist theory and pedagogy help us understand both math standards and indigenous teaching phenomenon.

A theoretical analysis on cultural related perspectives in terms of teaching and learning led to a new cultural model. Based on Sawyer's emergentist theory, cultural analysis is broken down to two-level analyses: societal-level and individual-level. The size of the group might contain individuals varying from one school to a district, a city, or a nation, or some combination of them. In Anthropology, two-level social property analysis has been debated for a long time. In this study, I adopted Sawyer's perspectives for resolving this controversial issue in the new model. That is, supervenience, multiple realizability, and wild disjunction will help explain the dynamic interactions between the two-levels (these definitions can be found in Chapter Four). Sawyer argued,

Social properties are supervenient on individual properties and yet not reducible to those properties (Sawyer, in press b, in press c). This account of emergence suggests that methodological individualists cannot argue a priori that all social properties and laws are

reducible to individual properties, relations, and laws, and that at the same time, methodological collectivists cannot argue a priori that a given social property is not so reducible. Whether or not a social property is reducible to individual properties, or a social law reducible to individual laws, is an empirical question that can only be resolved through empirical study. (p. 580)

In the new culture model, Confucianism and Taoism emerge as the fundamental beliefs and values in China, in contrast with behaviorism and individualism in the USA. A detailed analysis of this model can be found in Chapter 4 section 2. Some cultural elements were identified and examined in the actual teaching practice and interview data.

Data from the Constructivism-Culture and Actual Teaching Practice Survey (CCATPS) were analyzed both quantitatively and qualitatively. Qualitative analysis was adopted as the main part of this study.

The total score for the single subject in the first questionnaire was calculated in order to see teachers' espoused beliefs. The data from second and third questionnaire were analyzed by calculating the percentage of the subjects' responses for the certain items. These quantitative analyses demonstrated the tendencies of teachers to hold different constructivist theories (either radical or social) and manifest the relations between these tendencies and their cultural beliefs. The quantitative relations also help understand the change of teachers' beliefs in certain culture elements. That is, it is possible that Chinese teachers strongly advocate certain items that are identified as Western beliefs. And this can be explained by the new culture model because of its dynamic nature.

The actual teaching practice data and interview data are the main focus for data analysis. According to Figure 1.1, the actual teaching practice is influenced by teachers' understandings of constructivism. How and to what extent the teachers' beliefs affect their practice will be articulated when analyzing these two sources of data. According to the new culture model, the two-level culture analysis is suitable to analyze the data from the actual teaching practice.

The interview data were transcribed by three American undergraduate students. The hand analysis method was used for analyzing the interview data. I used codes to build description and themes, which were connected with indigenous culture models and constructivist beliefs. Five pre-determined themes were adopted for cultural elements' analysis. These themes are presented in the dimensions of Teachers' Role and Learning Process in the values questionnaire. A similar data analysis method was used for the Part IV data in CCATPS too. The concrete procedures for data analysis in my study are based on Creswell's guideline of qualitative data analysis (2005).

Table 4.4: The Overall Data Analysis

Types of data and the methods for analysis		Data sources	Aimed boxes in Figure 1	Purposes
Quantitative	Total score	Data from the first (reform) questionnaire	Box 4	The extent to which the teachers understand reform teaching and learning
	Percentages	Data from the second (constructivism) and the third (culture) questionnaires	Box 3 and Box 4	To look at the tendency toward both constructivist theory and indigenous culture
Qualitative	Theoretical	Constructivist, NCTM 2000/MOE 2001 Math Standard documents, and culture related literature	Box 1, Box2 , and Box 3	To create a model for cultural analysis, and to examine math standards documents through a constructivist lens
	Empirical	Data from interviews and teaching episodes (examples of practice)	Box 5	To understand the influence of indigenous culture in teachers' interpretations of Constructivism in each country
		Data from interviews and teaching episodes (teachers' perspectives on reform teaching and learning)	Box 4	
Mixed	A combination of qualitative and quantitative analysis (qualitative is the main part for analysis)			
Four final products	1. Teachers' understandings of constructivism 2. A new culture model 3. The empirical results of teachers' role 4. The empirical results of teachers' views of the learning process			

In summary, the data for this study were analyzed both quantitatively and qualitatively, and finally synthesized in using mixed methods. Table 4.4 provides a whole picture of the use of methodology and the possible results, which explain Figure 1.1 (see p.6) regarding the relationships between the data sources, the boxes in Figure 1.1, and the purposes.



## CHAPTER FIVE: EMPIRICAL DATA ANALYSIS

This chapter contains three sections. The first section aims to analyze the quantitative data of this study. The second section analyzes the significant cultural elements in China. And the third section is to analyze the significant cultural elements in the USA. The term “the American teachers” refers to American math teachers participating in this study. The term “the Chinese teachers” refers to Chinese math teachers participating in this study.

### Analysis of the Quantitative Data from China and the USA

In this study, three types of data are analyzed: 1) document data from constructivist theories, NCTM 2000/MOE 2001 Math Standards, and current cultural related perspectives in terms of teaching and learning, 2) data from Constructivism-Culture and Actual Teaching Practice Survey (CCATPS), and 3) interview data from both countries. The document data lead to a new cultural model and a theoretical evaluation of the NCTM 2000 and MOE 2001 Math Standards through the constructivist lens. The limitations and theoretical rigor of the NCTM 2000 and MOE 2001 Math Standards are analyzed based on constructivist theories in section one of this chapter. The new cultural model is analyzed theoretically in section two of this chapter. In this section, I focus on analyzing quantitative data, the data from three questionnaires in CCATPS.

### Participants and Instruments

Two samples of thirty middle-school math teachers from two mid-sized cities in both China and the USA participate in this study. Among Chinese teachers, thirteen are males, and seventeen are females. In the USA, ten males and twenty females participate. Ninety percent of the sampled teachers in each country report to have at least three-year experience in reform teaching.

All the teachers answered the CCATPS, which consists of four parts: Reform-Orientation Questionnaire, Teaching Style Questionnaire, Values Questionnaire, and Teaching Episode Writing. Each participant is paid for filling out the questionnaire. I thank SPSSI and College of Education at LSU for providing funds to my study. In this section, only the quantitative results from the first three questionnaires are reported.

#### The Results from the Reform Orientation Questionnaire

The Reform Orientation Questionnaire is an existing 20-item instrument used to assess K-8 school math teachers' beliefs toward reform using a five-Likert scale (see Appendix A). This questionnaire was developed by Ross, McDougall, Hogaboam-Gray, and LeSage with a high reliability ( $\alpha = .81$ ). The total score of the questionnaire ranges from 20 to 100. Seven items are reversed to indicate the positive relationship between the scores and the reform beliefs. That is, the higher the total score a teacher reports, the more reform-oriented the teacher tends to be. I set up the cutoff score as 60 because the range of the total score is from 20 to 100. A score of 60 or greater indicates that a teacher is reform-oriented. The mean and standard deviation are showed in Table 5.1.

Table 5.1: Descriptives from the First Questionnaire

	Mean	Min to max	Scores below 60	Std. deviation
China	73.7	57 to 90	1	7.26
USA	76.97	61 to 93	0	8.13

The mean 73.7 in Chinese sample indicates that Chinese teachers hold moderate reform beliefs regarding teaching and learning. The mean 76.97 in American sample indicates that American teachers hold moderate reform beliefs regarding teaching and learning. The only one score below 60 in Chinese sample is 57, in contrast to the minimum score 61 in American

sample. The above information confirms that most of the subjects hold reform teaching beliefs regarding teaching and learning.

### The Results from the Teaching Style Questionnaire

The Teaching Style Questionnaire is a constructivist theory-based questionnaire (see Appendix B). The theoretical foundations for the Teaching-Style Questionnaire are Radical Constructivism and Social Constructivism. Radical Constructivism adopted here is mainly based on von Glasersfeld's interpretation, in contrast to Vygotsky's theory of Social Constructivism. In addition, some perspectives are adopted that have extended our understanding of constructivist teaching and learning: Steffe's Zone of Potential Construction (ZPC) (1991), Simon's Hypothetical Learning Trajectory (HLT) (1995), Lesh and Yoon's evolving communities of mind (2004), Norton and D'Ambrosio's exploration of pragmatic differences between Steffe's Zone of Potential Construction (ZPC) and Vygotsky's Zone of Proximal Development (ZPD) (2008), and Kirshner's teacher-centered constructivist pedagogy (2008). In this questionnaire, the items on the left side are informed by Radical Constructivism, whereas the items on the right side are informed by Social Constructivism.

The Teaching Style Questionnaire is reported item by item in Table 5.2. The percentage in each cell is the ratio of the number of responses to each item choice to the number of total teachers. Teachers' strong radical (social) constructivist belief in a certain item is defined as the total percentages (strongly agree plus somewhat agree) in radical (social) aspect being over 70%. Table 5.2 shows that Chinese teachers hold very strong radical constructivist beliefs in item 5 (53.3% plus 26.7%), and strong social constructivist beliefs in item 9 (76.7% plus 16.7%) and item 6 (53.3% plus 26.7%). Chinese teachers' responses to item 5 indicate they strongly believe that teachers should provide learning tasks to students. Their responses to item 6 demonstrated their strong beliefs toward social constructivism regarding the design of tasks: The most effective tasks are designed in the process of teaching. Their responses to item 9 indicate their

strong social constructivist belief: Flexible thinking over deeply understanding concepts for the use of multiple solutions for the same problem. By contrast, American teachers hold very strong social constructivist beliefs in item 1 (40.0% plus 26.7%) and item 2 (46.7% plus 40%). This indicates that American teachers tend to believe that teachers must possess superior understanding of math knowledge and that developing cognitive dispositions is more important than helping students understand math content.

Table 5.2: Responses to Teaching Style Questionnaire in China and the USA

	Percentage (%)							
	Radical Aspects				Social Aspects			
Items	Strongly Agree		Somewhat Agree		Somewhat Agree		Strongly Agree	
	China	USA	China	USA	China	USA	China	USA
4	30.0	0	13.3	30.0	36.7	30.0	16.7	36.7
8	26.7	6.7	23.3	33.3	23.3	33.3	20.0	26.7
10	33.3	16.7	20.0	26.7	23.3	43.3	23.3	6.7
1	20.0	16.7	16.7	16.7	20.0	40.0	40.0	26.7
3	13.3	10.0	13.3	43.3	20.0	26.7	43.3	20.0
5	53.3	23.3	26.7	43.3	6.7	20.0	3.3	3.3
6	20.0	23.3	3.3	43.3	23.3	23.3	53.3	3.3
7	20.0	26.7	23.3	10.0	23.3	33.3	26.7	30.0
2	10.0	0	13.3	10.0	26.7	46.7	46.7	40.0
9	6.7	20.0	0	13.3	16.7	33.3	76.7	26.7

Note: The missing data are not included in this table.

### The Results from the Values Questionnaire

The values questionnaire is a seventeen-item cultural questionnaire. The theoretical foundation of this questionnaire is the indigenous culture model of teaching and learning in chapter four section two. This culture model perceives Confucian and Taoist culture as the fundamental roots in terms of teaching and learning in China, in contrast with Behaviorism and Individualism in the USA. In this questionnaire, the statements on the left side reflect Confucian or Taoist perspectives, while the statements on the right side stand for Individual or Behaviorism

perspectives. This questionnaire has ten dimensions: Value of Knowledge, Structure of Knowledge, Teachers' Role, Obligation of Society, Goals of Learning, Learning Process, Relation of Hierarchy, Moral Outlook, Attitude Relations, and Character of Students.

The items in the Values Questionnaire reflect the important cultural dimensions selected in both Eastern and Western cultures. Table 5.3 showed that Chinese teachers demonstrate very strong Eastern beliefs in five of the ten dimensions: Structure of knowledge, teachers' role, learning process, moral outlook, and attitude relations. Likewise, American teachers hold very strong Western beliefs in the following dimensions: Relation of hierarchy, Learning process, Structure of knowledge, and Value of knowledge.

In the values questionnaire, the two statements in each item are treated as cultural elements in Eastern and Western culture, respectively. I select five cultural elements for quantitative analysis. These cultural elements comprise two dimensions: teachers' role and learning process. The reason for selecting these five cultural elements is that they have been chosen as predetermined themes in qualitative data analysis. The quantitative results will serve as basic information for the further qualitative analysis.

In Table 5.3, the responses from over three fourth items are toward Eastern values. Chinese teachers in the items 2, 6, 8, and 13 demonstrate a tendency to Western values. Teachers' responses to some certain items reflect changes of some cultural elements. That is, Chinese teachers tend to select Western values, and American teachers tend to select Eastern values in those items. For instance, Chinese teachers demonstrate a great tendency toward Western values in item 8 (26.7% vs. 80%) and item 6 (13.47% vs. 86.7%). This phenomenon suggests that Chinese math teachers have been influenced in their cultural values by China's new math curriculum reform. For instance, the Western cultural value in item 8 is highly recommended in China's MOE 2001 Math Standards. That is, Chinese math curriculum overwhelmingly emphasizes teaching for all students. The accumulated percentages in item 6

(13.4% vs. 86.7%) illustrate Chinese teachers tend to advocate “adapted” rather than “emulated” for model persons. Switching to the Western values in item 6 can be explained by the effect of a long educational reform movement in China.

Table 5.3: Chinese Teachers’ Responses to Values Questionnaire

		Percentage (%)			
		Eastern Values		Western Values	
Dimensions	items	strongly agree	somewhat agree	somewhat agree	strongly agree
Value of knowledge	1	80.0	10.0	0	6.7
	2	16.7	13.3	30.0	36.7
	3	56.7	23.3	13.3	3.3
Structure of knowledge	12	53.3	33.3	10.0	3.3
Teachers’ role	5	46.7	23.3	16.7	13.3
	11	73.3	23.3	0	0
Obligations of society	16	33.3	23.3	36.7	3.3
	8	10.0	16.7	13.3	60.0
Goals of learning	15	76.7	20.0	3.3	0
	4	26.7	20.0	26.7	16.7
Learning process	9	43.3	16.7	20.0	13.3
	10	60.0	26.7	10.0	0
	14	66.7	30.0	3.3	0
Relation of hierarchy	6	6.7	6.7	26.7	60.0
Moral outlook	7	53.3	36.7	6.7	0
Attitude relations	17	73.3	23.3	0	3.3
Character of students	13	6.7	16.7	26.7	40.0

Note: The missing data are not included in this table.

In Table 5.4, American teachers tend to highlight Eastern values in items 11 and 17. In item 11, 83.3% of American teachers advocate the Taoist ideas of variety and balance for their pedagogy. In item 17, 63.3% of American teachers believe that positive and negative co-exist in all things. Advocating Eastern beliefs in item 11 indicates that American teachers have changed their behaviorist teaching beliefs due to the influence of the NCTM 2000 Math Standards.

Multiple ways for teaching are highly recommended by the Standards—this accords with fundamental beliefs in Taoism. Both Chinese and American teachers advocate Eastern beliefs in item 11. The different understandings of this belief will be discussed in the qualitative data analysis.

Table 5.4: American Teachers' Responses to Values Questionnaire

		Percentage (%)			
		Eastern Values		Western Values	
Dimensions	Items	strongly agree	somewhat agree	somewhat agree	strongly agree
Value of knowledge	1	23.3	20.0	50.0	6.7
	2	10.0	10.0	53.3	26.7
	3	10.0	13.3	53.3	23.3
Structure of knowledge	12	3.3	0	60.0	36.7
Teachers' role	5	6.7	6.7	56.7	30.0
	11	50.0	33.3	16.7	0
Obligations of society	16	26.7	30.0	36.7	6.7
	8	3.3	13.3	46.7	33.3
Goals of learning	15	20.0	36.7	23.3	20.0
	4	16.7	10.0	53.3	10.0
Learning process	9	0	0	66.7	33.3
	10	0	20.0	56.7	23.3
	14	23.3	16.7	43.3	13.3
Relation of hierarchy	6	3.3	6.7	50.0	40.0
Moral outlook	7	23.3	26.7	46.7	3.3
Attitude relations	17	43.3	20.0	26.7	10.0
Character of students	13	10.0	16.7	40.0	30.0

Note: The missing data are not included in this table.

The data from the two dimensions in Table 5.3 and Table 5.4 are drawn for the further analysis. The two dimensions are teachers' role and learning process. Teachers' role dimension contains two items: item 5 and item 11. Accumulated percentages are used in data comparisons. An accumulated percentage is the sum of the two percentages either from Eastern or from Western values scales. For instance, Chinese teachers' responses to Eastern values in item 5 were

46.7% for strongly agree and 23.3% for somewhat agree in Table 5.3. The accumulated percentage was 70%.

The accumulated percentages in the Chinese sample (see Table 5.5) in item 5 are 70% for Eastern values and 30% for Western values. This means that Chinese teachers believe that providing hints are more valuable than encouraging students or reframing tasks. In contrast, American teachers hold an opposite belief in item 5: 13.8% versus 86.7% for the accumulated percentages. The accumulated percentages in item 11 are 96.6% for Eastern values and 0% for Western values in Chinese sample, compared to 83.3% versus 16.7% in American sample. This indicates that both Chinese and American teachers hold that variety and balance for pedagogical balance are far more important than a single well-chosen method.

Table 5.5: Comparison of Accumulated Percentages on Teachers' Role

		Eastern values	Western values
Item 5	China	70%	30%
	USA	13.8%	86.7%
Item 11	China	96.6%	0
	USA	83.3%	16.7%

The missing data are not reported in this table

The learning process dimension included three items: item 9, item 10, and item 14 (see Table 5.6). The accumulated percentages in item 9 were 60% for Eastern values and 33.3% for Western values in Chinese sample, compared to 0 versus 100% in the American sample. This means that most Chinese teachers believe that students' mental struggle is more important than the sequenced instruction for the learning progression, while all American teachers deny mental struggle for the learning progression. Chinese teachers' responses to item 10 are 86.7% for Eastern values and 10% for Western values, in contrast to 20% for eastern values and 80% for Western values in American samples. This indicates that Chinese teachers believe that being



receptive to text and teachers is more important than expressing ideas for understanding, while American teachers favor the belief that expressing ideas can gain understanding.

Table 5.6: Comparison of Accumulated Percentages on Learning Process

		Eastern values	Western values
Item 9	China	60%	33.3%
	USA	0	100%
Item 10	China	86.7%	10%
	USA	20.0%	80%
Item 14	China	96.7%	3.3%
	USA	40%	56.6%

In Table 5.6, the accumulated percentages in item 14 are 96.7% for Eastern values and 3.3% for Western values in the Chinese sample, compared to 40% for Eastern values and 56.6% for Western values in the American sample. This means that Chinese teachers believe that the purpose of reviewing in the learning process is to consolidate math knowledge rather than to gain proficiency, while American teachers hold a moderate position between Eastern and Western perspectives.

### The Significant Cultural Elements in Qualitative Data from China

In this study, qualitative data are drawn from three sources: 1) documents to constructivist theories, NCTM 2000 and MOE 2001 Math Standards, and current cultural-related perspectives in terms of teaching and learning, 2) teaching episodes from CCATP survey, and 3) interview transcripts from both countries. The data in the first source have been analyzed in section one. In this section, the interview and teaching episode data will be the focus of data analysis. Five cultural elements are selected from values questionnaire as predetermined themes for the qualitative analysis. These cultural elements belong to the two cultural dimensions in the values questionnaire: teachers' role, and learning process. Quantitative data showed that Chinese teachers demonstrated strong tendency toward Eastern values in the five predetermined themes.

In addition, one emerging theme in both interview and teaching episode data is also identified as “emulative teaching”. The emergent theme is drawn by looking at overall patterns in the qualitative data (Teddlie & Tashakkori, 2009, p. 252). Hereafter, the six Chinese interviewees are abbreviated as CT1, CT2, CT3, CT4, CT5, and CT6. Researcher is abbreviated as RE. The teaching episodes are coded in random order from 1 to 30.

Teachers’ role and the learning process were selected as the two fundamental aspects in NCTM 2000 and MOE 2001 Math Standards, indigenous cultural model, and constructivist theories for qualitative data analysis. The different interpretations from constructivist theories and from indigenous cultures on these aspects are used for the comprehensive analysis. The similar ideas of these two aspects in NCTM 2000 and MOE 2001 Math Standards are demonstrated too.

### Teachers’ Role

A common claim about teachers’ role in both NCTM 2000 and MOE 2001 Math Standards is that a teacher should be a facilitator, a co-participant, and a class organizer. In the indigenous culture contexts, teachers’ role is interpreted in different ways. For example, in the values questionnaire (see chapter three), I identify two cultural elements, teachers’ support for concept learning and pedagogical balance, in the indigenous culture contexts. The first cultural element is manifested as “providing hints toward solution” in the Confucian learning tradition and as “encouraging students or reframing tasks” in the individualist and behaviorist tradition. The second cultural element is demonstrated as “variety and balance” in the Taoist tradition and as “selecting a single well-chosen method” in the behaviorist tradition. In constructivist theory, I identify teachers’ role in the following five aspects, and each aspect has two interpretations from radical constructivist and social constructivist perspectives. The five aspects include the following: 1) teachers’ and students’ roles in knowledge production, 2) facilitating students’ learning, 3) the source of task, 4) the design of tasks, and 5) open-ended and manipulatives tasks.

The cultural elements regarding teachers' role in qualitative data are summarized and analyzed as follows:

#### Teachers' Role—Providing Hints (70% vs. 30%, see Table 5.5)

In quantitative data, 70% of Chinese teachers advocate that providing hints is a common method in their teaching. Providing hints is an important cultural element in Confucian teaching. Chinese teachers provide extensive evidence in the interviews and the teaching episodes toward the Confucian cultural element: Providing hints for their students when teaching. The following interpretations demonstrate Chinese teachers' understandings of the purpose of providing hints. Chinese teachers also clearly state when the hints should be provided and what kind of hints should be given. Some Chinese teachers believe that the hints should be categorized in different levels in the learning activities.

#### The Purpose for Providing Hints

Chinese teachers perceive the main purpose for providing hints is to decrease the difficulty of the math concepts. That is, to help students understand math concepts, teachers should figure out the difficulty of the concepts. The hints serve as a means by which students are able to have mental struggles. CT1 has explicitly claimed his perspective in the interview:

RE: Before the 2001 math curriculum reform, math teachers focused on decreasing the difficulty of math concepts. For example, teachers usually intentionally designed a stumbling block and then they provided hints to guide students' learning. In the new curriculum, is there any change on this aspect?

CT1: I don't think there is any change on this point. Teaching is the process that the difficult math concepts can be broken at the different levels of understanding. Any types of designs, for instance, lecture, experiment, or manipulatives, should focus on seeking the ways to reduce the difficulty of math concepts. Teachers must provide hints for students after their mental struggle—by this way, students can understand.

#### When Chinese Teachers Provide Hints in Class

Most Chinese teachers claim that the hints must be provided when students feel uncomfortable in the class. They are able to understand students' feelings by observation. In the

interview data, four of the six math teachers explicitly state that they would consider providing hints when they observe the discomfort feelings from their students.

CT2: You can observe [students] faces, when you see their eyes are pleased, they can follow you. Some students frown; some are lost; and some don't pay attention—I know they don't understand the material. I will say, "I know you don't understand, please recall... or to see the connection between ....or how to lead this conclusion?"

### The Ways Chinese Teachers Provide Hints and the Levels of Hints

For CT1, Analogy is the main technique to provide hints in his classroom. Moreover, he believes that this technique can perfectly serve as a bridge to connect students past experience and the new knowledge.

RE: When you ask students to think about a problem, if it is too simple, the students will immediately respond; if it is too difficult, no students can follow you—as a result, students stop thinking. I recall that teachers should make students in a puzzle state in my past teaching experience. Do you have any ideas to share with me?

CT1: You talk about very difficult things in the class. Providing a situation [to students] needs time. So I usually use analogy to engage my students' thinking. For example, when I teach equilateral triangle, I do not directly present the properties of equilateral triangle, I ask students to recall isosceles triangle's properties first, and then ask students to think about equilateral triangle's properties and to see if isosceles triangle's properties are suitable for equilateral triangle. I also guide them to find new properties when students cannot continue.

Hints can be provided by demonstration. For example, in actual teaching practice data, a Chinese teacher recalls how to teach division of monomial. Instead of introducing rules and definitions, she asks students to review multiplication of monomial first, and then she intentionally demonstrates the following expressions on the blackboard:

Please fill out the parenthesis using the knowledge you have already known:

1)  $a^5 \cdot ( ) = a^7$

2)  $m^3 \cdot ( ) = m^8$

3)  $( ) \cdot a^3 = a^4$

4)  $( ) \cdot 5^n = 5^{m+n}$

$$5) (-a)^7 \cdot ( ) = (-a)^{10}$$

After you finish the above problems, please think about the following questions with awareness of the relations between multiplication and division.

$$1) a^7 \div a^5 = ( )$$

$$2) m^8 \div m^3 = ( )$$

$$3) a^4 \div a^3 = ( )$$

$$4) 5^{m+n} \div 5^n = ( )$$

$$5) (-a)^{10} \div (-a)^7 = ( )$$

(Teaching episode 2)

Students can receive hints from the first group of the problems. This teacher asks her students to explain how and why they get the answers to make sure if they understand the reverse relationships.

Chinese teachers also notice that hints should be prepared at different levels. That is, gifted students only need “subtle hints” (“点拨” in CT1’s interview); average students need “hints” that are carefully designed in teachers’ lesson plans; students with lower achievements need detailed hints with exercise problems. Nevertheless, most teachers’ focus is on the average students.

Due to the emphasis on all students in the MOE 2001 Math Standards, Chinese teachers provide more hints in class for students. This point is reflected in CT3’s interview:

CT3: If a few students understand, I would say that this is a difficult problem. I will give them some hints and ask them to think. I will continue to give them hints until most of the students understand.

Hints can be either predetermined in lesson designs or emergent from the situations. An inexperienced teacher is more likely to use predetermined hints that can be emulated from key teachers’ lesson plans. In contrast, experienced teachers can change their predetermined hints in their lesson plans instantly. For instance, CT1 says, “I do not always teach my students based on lesson plans, sometimes I discard the whole lesson plan and instantly come up with new

problems—I am seeking the best feelings for both my students and me, not too difficult and not too simple.”

The above description of providing hints does not summarize all techniques in this cultural element in China. As a cultural tendency, Chinese teachers use it either consciously or unconsciously in their classrooms. Experienced teachers possess better understanding than new teachers regarding how and when the hints should be provided. Hints can help students engage with the learning tasks. Hints are likely to be well designed for average students in Chinese teachers’ teaching.

Teachers’ Role—Making Balance and Variation (96.6% vs. 0, see Table 5.5)

The quantitative data discussed earlier show that 96.6% of Chinese teachers advocate the perspective of making balance and variation. The ideas of making balance and variation are key features in the Taoist tradition.

In this study, six themes regarding this cultural element have emerged from the qualitative data: 1) the balanced perspective between open-ended problems and closed problems, 2) the balanced perspective between one problem with multiple solutions and one problem with a single solution, 3) the balance between manipulatives and non-manipulatives, and 4) the balance between conceptual understanding and skill acquisition.

#### The Balanced Perspective between Open-ended Problems and Closed Problems

Chinese math teachers in this study demonstrate a strong cultural tendency toward making balance and variation when interpreting the use of open-ended problems and closed problems. Taoists believe that open cannot be separated from closed, and multiple cannot be separated from single. In the dynamic process, open should go toward close, and the multiple should go toward single, and vice versa. These beliefs implicitly lead to a unique way to perceive and to use open-ended problems and closed problems. Influenced by Taoist beliefs, Chinese teachers do not advocate that open-ended problems can be randomly open. In interview data,

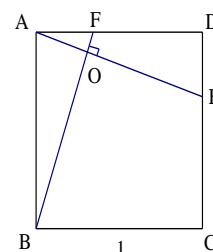
CT1 introduces his open-ended problem practice. He intentionally asks students to use sorting method to solve open-ended problems.

RE: What is your opinion on open-ended problems?

CT1: Adding some open-ended problems in the new textbook is a big change in the new curriculum reform. However, students have difficulty solving this kind of problems. So I only give them a short time for free thinking, and then I guide my students to use sorting strategies to solve certain open-ended problems.

Another example of using open-ended problems as a learning strategy and conceptual understanding is from a Chinese teacher's teaching episode. She designs a problem with three possible variations. In class, she gives variation one, and then she expects her students to find the other two. The teaching episode is as follows:

I design a problem with three variations. The problem is as follows: In square ABCD, E is in CD, and F is in DA,  $AE \perp BF$ , prove:  $AE = BF$ .



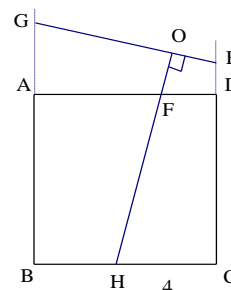
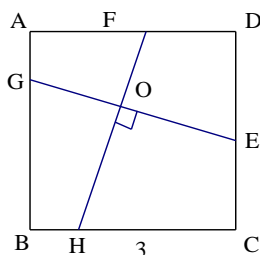
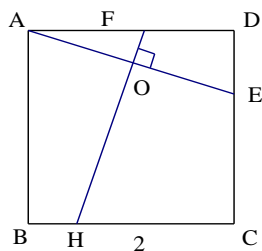
After lecturing on this problem, I will give students one problem based on this one. And I will guide them to solve it. I expect my students to find the second one and the third one.

- 1) If BF is parallel to HF (Keep HF perpendicular to AE),  $HF = AE$ ? [note: this teacher's statement is unclear. Here she refers to the second and third diagrams where point F moves from the left to the right]
- 2) If AE is parallel shift down to GE (Keep GE perpendicular to HF),  $GE = HF$ ?
- 3) If CE and HF intersect at O, and O is out of the square, is ....

In class, when I ask students to come up with the new problems, no one presents a problem like 2) or 3). For example, two students write the new problems as follows:

(1) If E, F, G, H in the sides of ABCD,  $EG = FH$ , then ABCD is square.

(2) If E, F, G, H in the sides of square ABCD,  $EG = FH$ , then  $EG \perp FH$ .



(Teaching episode 15)

In this case, the teacher selects the two problems from students' responses, and asks all students to make judgments if the proposition is true by giving a counter example or providing proofs. In fact, there are many types of problems that students come up with. However, the teacher does not encourage students to present these problems. Instead, she only selects two problems to ask students to discuss in class. She takes an open-closed style toward conceptual understanding, rather than thinking flexibility. This open-closed style is consistent to the ideas of balance and variation in Taoist tradition. Open and close playing together can make teaching effective.

#### The Balanced Perspective between One Problem with Multiple Solutions and One Problem with Single Solution

Chinese math teaches in this study contend a balanced perspective between one problem with multiple solutions and one problem with single solution. That is, they do not believe the more solutions the problem possesses, the more values the problem has for their teaching. Taoists believe that multiple cannot be entirely separated from single. Consequently, they do not have a preference to select math problems with multiple solutions, although MOE 2001 Math Standards recommend multiple ways to solve math problems. They merely adopt one problem with multiple solutions in their classroom with focusing on conceptual understanding rather than divergent thinking. For instance, CT1 believes that finding the similar methods is not the final purpose for his students' learning.

CT1: I advocate one problem with multiple solutions. However, I do not encourage students to find many similar methods to solve the same problem in class. If a problem has five different solutions, I encourage students to use one or two methods. For instance, corresponding angles, alternate interior angles, and same side interior angles can all prove that two lines are parallel. I do not encourage students to know all these methods as long as they know the existence of these methods, and that they can use one of them. If a problem can be solved by both geometric method and algebraic method, I highly recommend my students to understand and master them; it makes a difference.



### The Balance between Manipulatives and Non-manipulatives

Manipulatives versus non-manipulatives is identified as the third theme in making balance and variation. Manipulatives are highly recommended for use in the reform teaching. However, Chinese teachers do not advocate overemphasizing the use of manipulatives.

Chinese teachers believe that manipulatives should be balanced with non-manipulatives such as lectures, explanations, and students' excises. They state three reasons for keeping this balance. The first is that they do not believe manipulatives benefit the simple problem learning. This point is supported by CT1's interview:

RE: Do you often use manipulatives in your class?

CT1: Whether or not to use this method [manipulatives] depends on students. New curriculum emphasizes the concrete operations. In fact, it is not necessary to use it if the problem is too simple. It is enough to give them an explanation. For example, I can ask students to draw a triangle with three given restrictions to see if this triangle is unique. If it is unique, then its shape and size must be congruent. I can just explain this instead of letting students actually do the activity.

The second reason to make balance between manipulatives and non-manipulatives is the time constraints. Chinese middle school students must possess high skills and understandings for the state-level high school entrance examination. Teachers need to help them deeply understand math concepts in class. For example, in CT1's class, students are directly given the data by him when they learn statistics and probability. CT1 will explain the procedures for data collection. After that, students will do data analysis.

The third reason for making this balance is that teachers question what the students have learned in the activity. Chinese teachers tend to adopt teaching methods for conceptual understanding. They do not like to help students acquire dispositions. For instance, CT2 doubts in the interview, "Students would be very happy to explore on their own, but once they start playing, I can hardly stop them. When their excitement becomes hindered, some students will not pay attention to learning."

## The Balance between Conceptual Understanding and Skill Acquisition

Balance perspectives were not stated explicitly in China's math standards. However, the fourth aspect, conceptual understanding and skill acquisition, are balanced well in Chinese teachers' teaching. Before 2001 math educational reform the math community emphasized both understanding basic knowledge and acquiring basic skills. These "two basics" have been identified as one of the features of Chinese math education (Zhang, 2004). The Chinese 2001 new math curriculum reform overwhelmingly emphasizes conceptual understanding and dispositions. Although "two basics" are mentioned in MOE 2001 Math Standards. Chinese teachers express a middle-way toward harmonizing this reform belief with "two basics." This middle-way demonstrates a balanced perspective between conceptual understanding and skill acquisition. For instance, if they find that reform methods adopted from the new curriculum do not help students master the skills, they will return to use "old methods" in which the "two basics" determines how to assign students problems for practice, as CT3 states in the interview:

RE: Some teachers claimed that the new curriculum affects students' test scores; What are your strategies to improve students' skills when teaching new curriculum?

CT3: This is contradictory [between enhancing test scores and engaging students in activities]. When I find students cannot get good test scores in the quiz, I will assign them more math problems for practice. And this is supported by principals—we have weekly practice time periods for the mathematics. So I adopt some new elements from new curriculum for teaching, and use "old methods" for students' exercising.

### Discussion of Teachers' Role

As discussed above, providing hints and making balance and variation are important cultural elements in Chinese math teaching. Chinese teachers hold very strong beliefs on these cultural elements. The purpose of providing hints is to reduce the levels of difficulty in math learning. Through this method, students are able to engage in the learning tasks with a moderate level of mental struggles—a Confucian learning belief in the Chinese learning tradition. Making balance and variation is manifested in the five aspects in this study: 1) the balanced perspective

between open-ended problems and closed problems, 2) the balanced perspective between one problem with multiple solutions and one problem with single solution, 3) the balance between manipulatives and non-manipulatives, and 4) the balance between conceptual understanding and skill acquisition.

The two cultural elements (providing hints and making balance and variation) are either used consciously or unconsciously by Chinese teachers in their teaching. For instance, different teachers might prefer to use different methods to provide hints. Indeed, no strict and well-documented methods are used that state how to provide hints. The Confucian learning tradition forces Chinese teachers to find ways for the indirect teaching in order to engage students' thinking. The ideas of balance and variation are rarely discussed by Chinese math researchers or math educators, although these ideas are implicitly applied in the math teaching.

The claims that Chinese teachers use some of the techniques we discussed above for providing hints and making balance and variation do not mean that only Chinese teachers use these techniques. Rather, we can find American teachers sometimes also use similar techniques for their teaching. The differences are due to the cultural dispositions that the teachers have acquired in their indigenous cultural context. For example, Teaching Episode 2 can be either used for providing hints or for behaviorist teaching. If teachers do not possess a strong disposition toward students' understanding, they would go in a behaviorist direction rather than provide hints. Another example is the balance between lectures and practices that are used in both Chinese and American classrooms. The same teaching format can go in different directions if teachers use lecture for demonstration, and use practice for gaining proficiency, or teachers use lecture for conceptual learning, and practice for consolidating math knowledge.

Chinese teachers' responses concerning the teachers' role in the values questionnaire are consistent with the items 1 and 5 in the constructivist questionnaire. In item 1, 60% of Chinese teachers advocate a social constructivist belief that the teachers' role is to guide students'

learning based on their superior understanding of the content. This indicates that Chinese teachers believe that teachers' superior understanding is a prerequisite for providing hints and making balance among the methods and strategies they use in their classrooms. In item 5, 79% of Chinese teachers hold a radical constructivist belief that teachers should provide tasks geared to the content to be learned. This is consistent with providing hints and making balance and change.

### Learning Process

A common claim about the learning process in NCTM 2000 and MOE 2001 Math Standards is that students' learning should be toward understanding, and students should learn math on their own through participating in the learning activities. In the indigenous cultural contexts, the learning process is interpreted in terms of three cultural elements: 1) the ways of learning progression through mental struggle (Eastern) or through the sequenced instruction (Western), 2) the ways of understanding in learning either receptive (Eastern) or expressive (Western), 3) the purposes for reviewing in the learning process either consolidating (Eastern) or gaining proficiency (Western). In constructivist theory, the learning process contains three aspects: the effectiveness of lecture, the ontology of conceptual learning, and the origin of motivation. Radical constructivists do not deny the use of the lecture in classroom. By contrast, social constructivists believe that the lecture is not useful for students' understanding. In radical constructivism, ontology of conceptual learning comes from individual reflective practice, in contrast to social constructivists which focus on the acts of communication. Radical constructivists hold that students' motivation comes from the innate drive to understand, while social constructivists believe that students' motivation comes from their participation with others in social-directed activity.

The cultural elements regarding the learning process that is emerged in the qualitative data are summarized and analyzed based on three categories: mental struggle, receptive learning, and consolidating knowledge.

## Learning Process—Mental Struggle (60% vs. 33.3%, see Table 5.6)

The quantitative data show that 60% of Chinese teachers hold that students learn through mental struggle. In the interview data, Chinese teachers also emphasize that mental struggle is a very important aspect for students' math learning. The mental struggle can happen in class and out of class. In the interviews, many teachers express a perspective of math learning as closely related to mental struggle. They also believe that teachers should have mental struggle first in preparing their lesson plans in order to make students have mental struggle in class. In contrast to design tasks for students' in-class mental struggle, Chinese teachers also intentionally design homework problems for their students' long-time mental struggle.

Teacher CT5 claims, "There is no mathematics for fun. If the entire class time makes students feel very interested; there is nothing to learn." CT5 also contends that playing games does not benefit students' learning math. To CT5, real learning should make students thinking intensively.

Teacher CT3 condemns the 2001 math curriculum reform when talking about classroom activities. She believes creating students' activity will result in low test scores. To her, students should learn mathematics in a quiet place where they can reflect on their own work.

CT3: mathematics cannot be learned through participating in an activity. Students only struggle with math knowledge when they think quietly. You ask 50 students to participate in the activity, and different students learn different levels of math knowledge. It is only a slogan. In reality, I don't think it possesses the practical value.

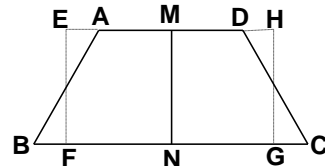
Teacher CT6 introduces an example of a teacher's mental struggle when making a demonstration lesson. The teacher first collects various problems that are related to his teaching topic. Then he needs to work on how to sort problems in different categories based on the difficulty of the problems. In this way, students will feel comfortable with the problems the teacher presents in class. Experienced teachers' design can put students in a thinking state, either not too simple or not too difficult.

CT6: teachers must spend time on doing difficult problems. Without this preparation, teachers cannot teach perfectly. When I solve a problem, sometimes I think about possible changes of this problem. In this way, one problem can generate four to five problems from simple to complex. It is necessary for teachers to have mental struggle with these problems, to reveal the relations among these problems, and to predict students' responses. I remember my mentor teacher spending ten days to struggle with how to use one problem generating multiple problems. His demonstrated lesson is to review the chapter on the circle. He collects over fifty problems regarding the circle chapter. The relations between problems are based on "the principle of monkey eating peach—jump to get the peach." Teachers accumulate these problems for years, and need to change when they teach a new class.

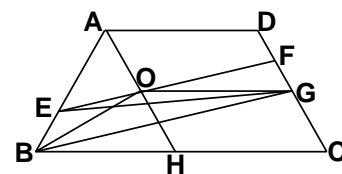
Teacher CT6 also provides a vivid teaching episode to show how he makes his students have mental struggle at home. He first assigns the problem that is very difficult to his students to solve. Then he asks students to work at home without giving any hints. In the next day, they work on that problem in class. The whole teaching episode is as follows:

Last semester when we learned the property of trapezoid, I asked a question to my students, "How many ways are there to partition a trapezoid into two shapes of equal area?"

Because a trapezoid is reflexive, many students thought about connecting the midpoints of the two parallel sides. I then asked, "Are there any other ways?" After a long lasting silence, one student said, "Can I use curved lines?" I said, "Of course!"



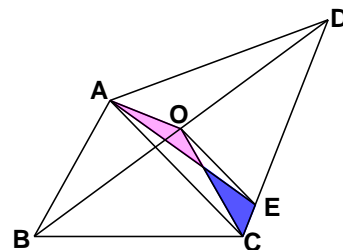
He partitioned the trapezoid into a triangle and a parallelogram and then cut each figure in half as shown in the figure (line BOG).



I then asked, "Can we modify this method so that we can use only one straight line?" I asked students to think about this problem after class.

The next day nobody solved it, so I explained my solution. "Let segment EF with E on AB and F on CD pass through O such that  $EF \parallel BG$ , then EG is our desired segment!" Students were confused, so

I explained, since  $EF \parallel BG$ , we have  $S_{\triangle BEG} = S_{\triangle BOG} \therefore S_{\triangle BEK} = S_{\triangle GOK}$ . Since  $S_{\text{五边形}ABOGD} = S_{\text{四边形}BCGO}$  we have  $S_{\text{四边形}AEGD} = S_{\text{四边形}BCGE}$ .



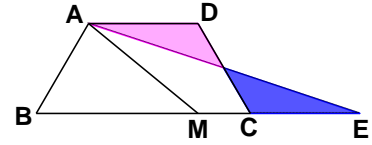
After a few minutes, the entire class cheered, "Intriguing!"

How come I didn't think about that?" I continued to bolster their intellectual engagement, "What if we replace the trapezoid with an arbitrary quadrilateral?"

The next morning, one student rushed into my office to show me his discovery. In quadrilateral ABCD, let O be the midpoint of BD. Obviously, line AOC partitions the quadrilateral into two equal areas. Let E be a point on

CD such that  $OE \parallel AC$ . Then AE is the desired line. I was excited about his enthusiasm. I highly praised him, "You must constantly reflect on the math knowledge. In this way, you can discover better methods and interesting facts."

(Teaching episode 12)



In teaching episode 12, teacher CT6 changes his routine teaching method—gradually providing hints for students' mental struggle. Instead of providing moderate learning tasks to his students, he assigns his students a difficult problem for homework without any hints. His students should have a long-time mental struggle at home. As they come back to school the next day, CT6 directly writes the answer on the board without any hints for his students. This teaching strategy leads to a cheerful moment in the classroom. This long-time struggle reflects a Confucian precept: "I will not open the door for a mind that is not already striving to understand, nor will I provide words to a tongue that is not already struggling to speak." (*Analects*, book 7, p. 66. Translated by Slingerland). After CT6's explanation, students are given a new problem to explore: What if we replace the trapezoid with an arbitrary quadrilateral? Some students have already known that CT6's method can serve as a hint for this new problem and make a perfect connection between the old and new problems.

Learning Process—Receptive (86.7% vs. 10%, see Table 5.6)

Quantitative data show that 86.7% of Chinese teachers hold that being receptive to the text and teacher is the primary path to understanding. This is consistent to the interview data. All Chinese interviewees advocate this belief either directly or indirectly. Three types of interpretations are identified in teachers' interviews.

Some teachers overtly question the ideas of cooperative learning. They prefer to use traditional Chinese teaching methods (e.g., lecture plus practice) in their classrooms. For instance, teacher CT5 cannot follow the reform idea that “Curriculum developers call for the new approach that teachers give students enough time for discussion or activity-based learning” (CT5’s interview). However, these teachers intentionally use cooperative learning activities in the demonstrated lessons to meet the criteria of reform teaching. Teacher CT4 in the interview gives us a clear picture on her teaching.

RE: Would you like to talk about your own perspectives on the new curriculum reform?

CT4: I don’t think there is a dramatic change in classroom teaching. To me, the big change of my teaching is the lesson preparation. I have to use new textbooks, and I also adopt problems from the old textbooks. Although sometimes I try to give students more time to think, I still dominate the class. For most teachers, the students’ activities are only provided intensively in the demonstrated lessons.

RE: Do you ask your students to have group discussion?

CT4: In my class, I mainly focus on thinking independently. I believe most of my students possess the ability of independent thinking. So I prefer to guide my students for their own learning. I think we cannot change our old beliefs quickly. I always feel it waste students’ time if I ask them to participate in an activity. For example, if I give them a difficult problem, they probably discuss half an hour without results. If I guide them to understand, I only need five minutes. However, if I conduct a demonstrated lesson, I will design tasks for students’ group discussions.

In the above conversation, CT4 clearly expresses her perspectives regarding students’ learning—“students possess the ability of independent thinking.” Under this learning assumption, she prefers to use “lecture plus practice” format, a common method in Chinese math teaching, to teach her students. The resistance of cooperative learning does not mean that she is not able to conduct lessons using reform methods. Rather, Chinese teachers demonstrate a cultural disposition toward receptive learning.

Some teachers claim that they strongly agree with reform ideas such as students’ participation in the learning activity. However, their interpretations of their concerns and their classroom teaching imply a strong Chinese traditional belief in their reform teaching.



RE: Do your students like to express their ideas in class?

CT2: My students do not like to express their ideas in class. I found students are less likely to raise their hands as they grew older.

CT1: Sometimes I try to give students time for free discussion. Their thinking is divergent—so many questions are raised by my students. You know fifty students may have fifty ideas. Sometime we cannot solve one of them. It is really time consuming.

Teacher CT2's response regarding students' participation implies that he does not switch his role as a co-participant in classroom. The phenomenon that his students are reluctant to raise their hands reflects that he has not found an appropriate way to help his students participate in the learning activity. CT1's frustration on his students' divergent thinking reflects the lack of skills to organize the classroom discussion. Although these teachers claim that they advocate reform teaching, we can imagine that expressing students' ideas would not be highly encouraged in their teaching.

Some teachers claim that asking students to participate in a classroom activity is to assess if students understand teachers' lectures. They usually ask students sitting in their seats to speak. These teachers believe they successfully conduct reform lessons with students' participation. However, their interpretations indicate they prefer to accept the idea of "being receptive" rather than the idea of "being expressive."

CT6: Students can sit in their seats asking questions any time, if they do not understand. When I ask students speaking in their seats, what I am looking for is the mistakes they made. I will directly discuss their misunderstandings. Sometimes students quickly speak out the answers without thinking; I will tell them this is not a good habit.

Learning Process—Consolidating Knowledge (96.7% vs. 3.3%, see Table 5.6)

Quantitative data shows that 96.7% of Chinese teachers hold that reviewing and reflecting on knowledge that students have learned leads students to gain new knowledge and understanding, rather than leads to greater proficiency with skills. This perspective is reflected from both interviews and teaching episode data. In interview data, teacher CT2 introduces how

Chinese teachers help students prepare for high school entrance exams and how the purifying process can help students gain deeper understanding and proficiency. In actual teaching practice data, three teaching episodes are selected to show how Chinese teachers use their own teaching methods to consolidate math knowledge.

Teacher CT4 introduces the reviewing process that is not included in the regular class. This process will help students gain proficiencies with deeper understanding of math concepts. Teachers usually focus on mistakes that students made and have lecture during evening practice class (晚自习):

CT4: We have six regular classes per week. On Saturday we have one or two practice classes. We also have one or two evening practice classes (晚自习) per week. In addition, each week we have an 80-minute quiz. During the evening practice class, teachers usually spend half time lecturing on the mistakes from students' homework and half of the time for practice.

Teacher CT2 believes that doing math problem is the process of purifying one's thinking. Learning is not an incremental process, so extensive practice does not improve students' learning:

CT2: In fact, learning math is the process of purifying one's thinking. Solving one problem means that one understands how to solve the similar problems. I always tell my students that solving a problem should summarize key points underneath that problem. After you understand the ways to solve the problem, you should rethink it to try to find a simple way to solve it and the possible ways the problem might be changed. Some students solve many problems without enhancing their test scores: why? Because they do not have the purifying process. They do not think back and forth; do not seek the relationships among the problems. However, you can only teach the purifying process for the students with average or above average math ability. For students with lower math ability, they are not able to learn this process, even after you teach these students many times.

Based on these beliefs, CT2 shows us how Chinese teachers carefully design their reviewing content and problems for state-level high school entrance examinations. This is a typical case for most Chinese teachers who teach the ninth grade:

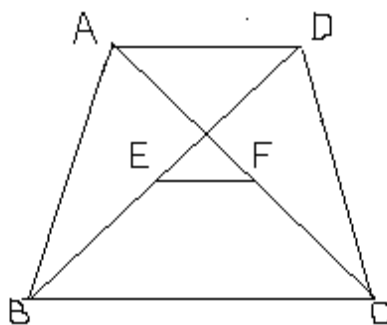
CT2: I have to spend six months to review all math knowledge. I first review math concepts unit by unit, for example, numbers and expressions, equations and system

of equations. The problems should be prepared gradually from simple to difficult. Students should experience the variation of the problems. After this step, I will guide my students to the comprehensive review.

The above interview data only give us general information regarding Chinese teachers' perspectives on consolidation and proficiency. To specify this consolidating practice, three teaching episodes are chosen for understanding the Chinese ways of reviewing math knowledge.

Teaching episode 25 provides a vivid picture on how Chinese teachers perceive proficiency and how to gain proficiency through understanding rather than through repetitive practice. This teacher guides students to discover a “semi-theorem” by which students gain a great proficiency. The teacher does not present the rules directly to the class. The students first experience the process to prove this problem. The proficiency has been gained after students' understanding—They only use 1 to 2 seconds to find the answers. The teaching episode called Semi-theorem is as follows:

Recalling my past teaching experience, I found the following teaching episode is very impressive. Last year I conducted a review class on median of a trapezoid. My students spent a long time to solve this problem: E, F denote the midpoints of the diagonals of trapezoid ABCD,  $AD=2$ ,  $BC=6$ , Find EF.



When I gave a lecture on this problem, I first guided them to use auxiliary lines, and then to prove. This is a complicated process. Reflecting on this problem, I found a pattern for this kind of problems. If students understand this pattern, they can use it as a bridge to solve more complex problems. If they encounter this problem as a fill-in-the-blank problem, they will quickly fill out the answer.

To help students get the pattern, I gave them a very similar problem:  $AD=4$ ,  $BC=8$ , find EF.

This time, students still used a long time (5 minutes) to get the conclusion. Although they can solve it, it was obvious they spent too much time on this problem. I asked them: “Please analyze the relationships between the data from the above two problems. I

encourage you to find a pattern.” Students talked to their neighbors, and a few minutes later, one student said, “I found the pattern, EF equals to half the difference between the two bases.” I am very glad, and further questioned: “Why, can you prove it?” After that, students only use 1-2 seconds to solve the similar problems. I call this general property a “semi-theorem.”

(Teaching episode 25)

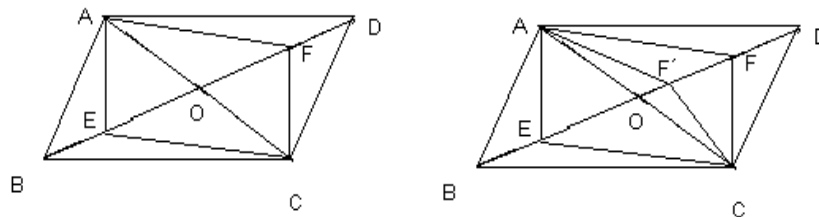
Teaching episode 1 shows how a Chinese teacher consolidates basic knowledge and basic skills through an open-ended problem and the counter-example. The teaching episode called Necessity for Parallelogram is as follows:

Focus: problems related to congruent triangles, parallelogram’s property and necessity

Difficulty of teaching: flexible application of necessity for parallelogram

Description of the part of the teaching process:

Example 2. In parallelogram ABCD, E, F on BD. What conditions can we add so that AECF must be a parallelogram?



Students answered in different ways: 1)  $BE=DF$ , 2)  $\angle BAE=\angle DCF$ , 3)  $AE\parallel FC$ , 4)  $AF\parallel CD$ , 5)  $\angle DAF=\angle BCE$ , 6)  $EO=OF$ , and 7)  $BF=DE$ .

Suddenly, some students asked whether  $AE=CF$  or  $AF=CE$  would work. Students started discussion.

Students concluded that if  $AE=CF$ , then “SSA” would not prove the congruence of  $\angle AEO$  and  $\angle CFO$ .

Teacher then gave a counterexample: if  $CF'=CF$ , then  $AFCF'$  is not a parallelogram.

Teacher: Under what condition can  $AE=CF$  guarantee AECF is a parallelogram?

Students: “HL (Hypotenuse-Leg)” [note: need to check the meanings]

Teacher: When can we have “HL”?

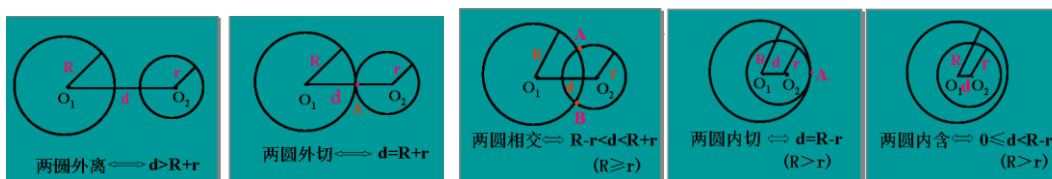
Students: When  $AC \perp BD$ , . . .

(Teaching episode 1)

In teaching episode 1, students review congruent triangles and the property and necessity of parallelograms. Students fully grasp the basic knowledge in the reviewing process. The proficiency is not gained from practice of the same type of problem. Rather, the teacher takes a reversed process to ask students working on an open-ended problem. The students should first add conditions, and then they need to prove. The counter example and HL condition help students gain insight of problem-solving skills. To some extent, the flexible application of necessity for parallelogram entails proficiency, flexibility, and conceptual understanding.

Teaching episode 11 demonstrates how a teacher helps students understand and memorize the positions of the two given circles. This teacher provides visual representations to help students grasp the relationships between the radii and the distance between the two centers. Moreover, the teacher creates a number line to illustrate  $0$ ,  $R-r$ ,  $R+r$ ,  $d$ , and how to use the segments to connect the previous figures. The ways to memorize the relationships among  $0$ ,  $R-r$ ,  $R+r$ , and  $d$  are purified. That is, students not only quickly recall the knowledge, but also they can explain the reason for having those relationships by recalling the previous figures. The teaching episode is as follows:

Summarizing the two methods to show the positions of two given circles



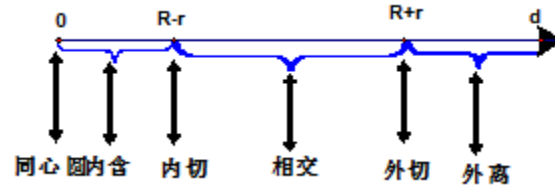
1. Number of intersections
2. Relationship between the radii and the distance between the two centers.

Combining numbers with shapes and using the computer to teach efficiently.  
Design figures so that students can visualize the relationships between  $r$ ,  $R$ , and  $d$ .  
Search for patterns, summarize conclusion:

I displayed pictures on the computer during my students' discussion. Students must pay close attention to the triangle formed by  $d$ ,  $R$ ,  $r$  when the circles are intersecting.

1) Externally non intersecting  $\Leftrightarrow d > R + r$ , 2) External tangent  $\Leftrightarrow d = R + r$ , 3) Intersecting  $\Leftrightarrow R - r < d < R + r$ , 4) Internal tangent  $\Leftrightarrow d = R - r$ , 5) One contained in another  $\Leftrightarrow d < R - r$ .

A simple way to remember this relationship:



Designing the figures and the number line is the highlight of this teaching episode. My past teaching experience told me that students had a hard time remembering these relationships. Using both figures and the number line makes memorization more clear and accurate.

(Teaching episode 11)

### Discussion of Learning Process

Chinese teachers demonstrate a great Eastern cultural tendency in the learning process. They believe that students should learn through mental struggle. They hold that being receptive to the text and teacher is the primary path to understanding. They also strongly believe that reviewing and reflecting on knowledge that students have learned leads students to gain new knowledge and understanding. These three interpretations in qualitative data are in accordance with quantitative data where the ratios of the responses between radical and social constructivism are 60% to 33.3%, 86.7% to 10%, and 96.7% to 3.3%, respectively. The third cultural element, consolidating knowledge in the reviewing process, is the highest one among the three cultural elements.

Students' learning through mental struggle is highly related to the cultural elements "providing hints" and "receptive learning." Specifically, for in-class instruction, to provide hints is to reduce the difficulty of the math concepts. This keeps students in a moderate state of mental struggle. Moreover, these two cultural elements lead to the belief of receptive learning. In the qualitative data, Chinese teachers have different voices regarding receptive learning. Some of them directly claim that students' learning should be receptive, rather than expressive; some of

them claim that students' learning should be expressive, a reform tone of learning. However, the teachers with a reform tone still conduct their lessons that benefit receptive learning rather than expressive learning as we demonstrated before. This phenomenon reveals that on one hand, holding beliefs of "providing hints" and "learning through mental struggle" becomes an obstacle for Chinese teachers to adopt the reform idea of expressive learning from Western culture; on the other hand, adopting an imported cultural element is a long-time assimilating process—Chinese teachers cannot achieve this in a short time. They do not possess the dispositions toward students' discussions and the sophisticated skills to shape students' activities.

Consolidating math knowledge is mainly manifested in Chinese teachers' review lessons. The review lessons are prepared for the evening practice classes (晚自习) and the reviewing classes for high school entrance examinations. As I analyzed in the qualitative data, Chinese teachers believe that reviewing math knowledge is very important for students' understanding math knowledge. This belief also implies that math understanding cannot be achieved in one time—a Confucian perspective regarding how to learn knowledge. Chinese teachers interpret reviewing math knowledge as a purifying process, a process to make students think clearly and logically. For example, the students can quickly finish a problem by memorizing patterns and knowing the reason for getting that pattern (e.g., teaching episode 4).

Chinese teachers' cultural tendencies both in qualitative data and the values questionnaire at the dimension of the learning process are consistent to their perspectives in the items 8 and 10 in the constructivist questionnaire. In the constructivist questionnaire, Chinese teachers' responses to item 8, ontology of conceptual learning, and item 10, the origin of motivation, are 50% to 43% and 53.3% to 46.6%, respectively, a slight tendency toward radical constructivism. That is, just over half of the Chinese teachers believe that individual reflective practice is more important than the acts of communication for conceptual learning. Over half of them also believe

that the motivations are from the innate drive to understand rather than students' social participation. Chinese teachers' responses to item 4, the effectiveness of lecture, are 43.3% to 53.4%, a slight tendency toward social constructivism. This is a little different from their cultural values with "receptive learning" and "learning mental struggle."

#### Emulative Teaching—An Emergent Theme in Chinese Data

Emulating a model person is rooted in the Confucian tradition. One of the core beliefs in Analects is the ideas of emulation. Confucius defined "gentleman" (Junzi) as an ideal person who should be emulated by others (Slingerland, 2003, p. 238). In this study, emulative teaching refers to a teacher's teaching that is emulated from model teachers or experienced teachers. Emulated teaching is manifested in both interviews and teaching episodes. All teachers in the interviews claim that experienced teachers' teaching is very important for new teachers to develop their teaching skills. For instance, CT6 can exactly state his mentor teachers' lessons when recalling how his mentor teacher prepared demonstrated lessons. CT5 and CT4 tell me that teachers in some schools used the same lesson plans to teach. These lesson plans come from experienced teachers. CT5 and CT4 strongly emphasize the role of research lessons and research groups in teachers' professional development. These lessons and group work truly reflect an emulative style.

RC: Would you like to introduce something regarding individual teacher's professional development?

CT4: Key teachers' demonstrated lessons are the main way to improve other teachers' teaching skills. Sometimes teachers conduct research lessons in turn, and the key teachers will make comments on how to improve these lessons.

RC: How do math teachers improve teaching skills in your school?

CT5: The first way is to learning from key teachers' demonstrated lessons. But I think the most important way is "the lesson plan co-writing" (集体备课) within a "teaching group" (教研组). New teachers must possess humble attitudes and show their respect to the key teachers. New teachers must go to their mentor teachers' classrooms to observe continuously.



Here “the lesson plan co-writing” means that all teachers who teach the same grade level get together discuss how to write the lesson plans. In this discussion process, the key teachers play a leading role in guiding other teachers. Emulative teaching is manifested in some teachers’ teaching episodes. Some teachers tend to explain their teaching in a strict Chinese teaching format.

There are no unexpected teaching events or real reflections in the teaching episodes. Rather, their teaching episodes indicate that their effective teaching means to develop standard lesson plans. The following case entitled “Factorization by taking out the common factors” shows this type of teaching:

Teaching Focus: The meaning of factorization and the method of taking out common factors

Teaching Difficulties: Finding the common factor and comparing the factorization of polynomials with those of integers

Teaching process:

1. Creating an environment leading to the new material

Fill in the blank with the knowledge of multiplication of polynomials

$$1) m(a + b + c) = \underline{\hspace{2cm}} \quad (2) (a + b)(a - b) = \underline{\hspace{2cm}}, \quad (3) (a + b)^2 = \underline{\hspace{2cm}}$$

Trial and explore:

$$(1) ma + mb + mc = \underline{\hspace{2cm}}, \quad (2) a^2 - b^2 = \underline{\hspace{2cm}} \quad (3) a^2 + 2ab + b^2 = \underline{\hspace{2cm}}$$

Observation and Conjecture: What is the difference between these two sets of equations?

Guide students to find: Group 1, Left is\_\_ Right is\_\_  
Group 2, Left is\_\_ Right is\_\_

2. Group discussion and exploration

Through the above exercise, discuss the concept of factorization in groups.

3. Examples, 1, 2, 3, 4
4. Exercise
5. Conclusion

(Teaching episode 17)

Teaching episode 17 superficially demonstrates a “sophisticated” way to develop lessons. However, it implies a lack of understanding in effective teaching. The teacher is not able to recall any significant moment or unexpected event in the classroom. Instead, he presents a regular teaching format that is typically adopted from experienced teachers. His teaching episode implies that he tends to emulate model lesson plans without real understanding of them. As a result, this emulation becomes a routine in developing his lesson plans. Some of his words such as trial, explore, observation, and conjecture used in this lesson are from reform curriculums. These words do not possess their original meanings from this teacher’s sense.

Another example for the emulative teaching is the teaching episode 8; it seems the teacher uses teacher-student conversation styles to state the lesson. Surprisingly, the students’ responses are exactly the same as this teacher wrote in the teaching episode. This teacher-student conversation styles were very popular in the 1990s in China. That is, some publishers were enthusiastic to publish key teachers’ lessons in this style. Many math teachers in China have emulated these teachings for a long time. The teaching episode is as follows:

1. Creating an environment leading to the new class

Teacher: Imagine a triangular grassland. How can you calculate its area?

Students: Measure the base and the altitude, then use the area formula.

Teacher: Good. The altitude is an important segment of a triangle. It is closely related to the area. But what is the altitude and how can we draw the altitude? Today we are going to learn the altitude, median, and angle bisector of a triangle.

2. Experiment and exploration

Teacher: What is the altitude?

Student: Draw the perpendicular from vertex A to side BC with D as the foot. AD is called the altitude.

Teacher: How do we translate this into the language of geometry?

Students: Because AD is the altitude of  $\triangle ABC$  ( $AD \perp BC$ , D is the foot),  $\therefore \angle ADB = \angle ADC = 90^\circ$

Teacher: How many altitudes does a triangle have? Draw them and observe their relationship.

Student 1: I drew three altitudes of an acute triangle. They coincide in the triangle.

Student 2: I drew an obtuse triangle. Two altitudes are outside the triangle, but the three did not coincide.

Teacher: Watch me draw an obtuse triangle using Geometry Sketchpad. Its three altitudes do coincide. This point is called the orthocenter.

Student: I drew a right triangle. Two altitudes are two of its sides. The three altitudes coincide at the vertex.

Teacher: Good. Now let's look at the following problem.

... ..

(Teaching episode 8)

The above two teachers' teaching episodes reflect the negative aspect of emulative teaching. The positive aspect of this kind of teaching is demonstrated in some teaching episodes where the teachers work toward emulating reform lesson designs. For instance, five Chinese teachers have developed activity-based lessons in the teaching episodes 3, 7, 10, 13, 22, although these kinds of designs are common in American teachers' teaching episodes. This cultural tendency will compromise the resistance from their indigenous culture regarding activity-based classes. Moreover, emulative teaching greatly enhances most of the new teachers' professional developments.

Emulative teaching is not explicitly advocated by Chinese math educators and math teachers. In the past 30 years, the imported culture regarding teaching and learning has heavily influenced Chinese math education. Creating new teaching methods and strategies are highly encouraged in China. As a result, Chinese traditional teaching beliefs such as Confucian and Taoist perspectives in teaching and learning are downplayed. To better understand emulative

teaching, I briefly introduce some important teaching experiments in math education in China in the following paragraphs.

A unique characteristic in teaching in the past 30 years in China was the innovative teaching strategies created by model teachers or by influential educational experiments. This was rarely mentioned by comparative researchers when they articulated Chinese ways of teaching and learning. In these teaching experiments, the big classroom size was rarely changed; the teachers' role was rarely changed dramatically; the changes usually occurred when adopting new culture elements (e.g., problem solving ability), or implicitly integrating Chinese fundamental beliefs into the curriculum. For instance, Gu, a math educator in Qingpu County, Shanghai, and now a professor, started his math teaching experiment in 1977. He summarized the following strategies for teaching—that became one of the most influential examples in China.

1. Using problems as a starting point for teaching;
2. Guiding students to develop exploratory activities;
3. Establishing variation in practice to raise the effectiveness of practice;
4. Summarizing to adopt into the knowledge structure; and
5. Modifying according to the fine categorization of teaching objectives.

(cited in Lopez-Real, Mok, Leung, and Marton, 2004)

Like Gu, many teachers in China (e.g., Weigang Sun, Qianxiang Zhao, Guimei Dou, Shusheng Wei) earned nationwide reputations through their teaching experiments. Many teachers in China are eager to emulate these model teachers' teaching. In China, learning a popular teaching experiment is highly supported by the school boards or education administrators. Emulating these experiments and teaching strategies truly reflected a Confucian tradition—a social cultural property that is implicitly embedded in the Chinese cultural context.

## The Significant Cultural Elements in Qualitative Data from the USA

In this section, the same five cultural elements I identified from the values questionnaire in the last section are selected as predetermined themes for the qualitative analysis. These cultural elements belong to the two cultural dimensions in the values questionnaire: teachers' role, and learning process. Quantitative data showed that Chinese teachers demonstrated a strong tendency toward Eastern values in the five predetermined themes. One emerging theme in both interview and teaching episode data is identified as "individual caring." The six US interviewees are abbreviated as AT1, AT2, AT3, AT4, AT5, and AT6. Researcher is abbreviated as RE. The teaching episodes are randomly coded from 1 to 30. The term "the American teachers" refers to American math teachers participating in this study. The term "the Chinese teachers" refers to Chinese math teachers participating in this study.

### Teachers' Role

The claim on teachers' role in both NCTM 2000 and MOE 2001 Math Standards is that a teacher should be the facilitator, cooperator, and class organizer. In the indigenous cultural contexts, teachers' role is interpreted in different ways. For example, in the values questionnaire (see Chapter Three), I identify two cultural elements in the indigenous cultural contexts: teachers' support for concept learning, and pedagogical balance. The first cultural element is manifested as "providing hints toward solution" in the Confucian tradition and as "encouraging students or reframing tasks" in the individualist and behaviorist tradition. The second cultural element is demonstrated as "variety and balance" in Taoist tradition and as "selecting a single well-chosen method" in behaviorist tradition. The cultural elements regarding teachers' role in qualitative data are summarized and analyzed as follows:

#### Teachers' Role—Encouraging Students or Reframing Tasks (90% vs. 10%, see Table 5.5)

The quantitative data show that 90% of American teachers believe that encouraging students or reframing tasks is more important than providing hints when teaching concepts. In

the interview and teaching episode data, American teachers demonstrate at least three ways to accomplish this goal: 1) Grouping students, 2) Encouraging students who have low scores, and 3) Providing real-life activities.

### Grouping Students

The first way, grouping students with different abilities, is a common strategy for American teachers to encourage their students. They usually put low-level students with high achievers to make sure that the low achievers can be encouraged by the high achievers, as a teacher writes in the teaching episode 9: “I was sure to place lower-level students with higher-achieving students to act as role models and peer instructors.” Some teachers also claim that electing a group leader will help students establish their confidence.

The groups that were formed consisted of one high achiever, one average achiever, and one low achiever. As a result of their efforts to help others, group leaders reinforce their own knowledge and skills, which in turn builds their self-confidence and self-esteem. (Teaching episode 1)

By contrast, some American teachers group students homogeneously. They put same-achievement students in the same group, and then they assign students different tasks. Low achievers receive basic tasks, and high achievers are assigned more challenging tasks. These teachers believe that students can do their best to engage each student equally and confirm understanding by this grouping strategy. The students usually are divided into three or four levels. The following teaching episode shows a three-level grouping strategy:

During a 6<sup>th</sup> grade lesson on fractions and percents and also taking percents off sale items, students were required to participate in a shop-a-thon. The students were to select three items from a choice of four different locations of shopping stores on posters that were placed around the room. The students were paired up based on their academic performances. Based on their academic performances, the students were given certain instructions relating to Tier1, Tier 2, and Tier 3. Tier1's had to find the percent off. Tier 2 had to find the percent off plus the price after percent off. And Tier 3, the percent off, price after percent and then take an additional percent off. This teaching episode was very successful because students were placed in the areas of performance were given a good instruction and were given higher order thinking questions. The students also enjoyed the lesson because it was based on real-life situation. The entire class was involved and engaged. (Teaching episode 14)

## Encouraging Low-Performed Students

The second way, encouraging students who have low scores, also emerged in the interview and in the teaching episode data. When teachers walk around the classroom, they pay attention to the low achievers:

I saw students who are academically weak, assist other students in exploring this concept because the lesson enabled them to construct squares and determine square roots through a visual representation of this theorem.  
(Teaching episode 6)

The low achievers also are encouraged to ask questions in class. Teachers either help them figure out what is right and wrong or ask other students to give a correct answer. Instead of providing hints, American teachers tend to give direct instruction or reframe the problems:

AT1: The lower learner is going to hopefully ask questions, and we the teachers know how they scored before, so when we are giving our demonstration, we zero in on that child, and I'm asking them questions like "Did you get it? Do you understand it?" and then I'll say, "Now what does this mean?" And, "when we did this, what did we miss?" And then I make the little rhyme. . . . So I just ask them questions really, and then we just kind of do it together, is the way I do it. (the interview)

Dalvin was the first student that I asked, "What are the two factors that every number has?" He knew that 1 is a factor of every number but was hesitant to give the other factor. I considered this to be a critical moment so I **restated** the question. After Dalvin gave the correct answer, I decided to rotate to all groups and ask them the same question to ensure understanding.  
(teaching episode 1)

Some teachers demonstrate a sophistic skill for all students' understanding. They ask students to talk about the process, rather than the correct answers in group discussion. They have noticed that giving a correct answer is not a guarantee for understanding, specifically, for the low achievers. The following interview conversation reflects the teacher's effort to switch students' attention from correct answers to the process.

RE: Do you think your students can express their ideas thoroughly in groups when you pair them for discussion?

AT2: When they share their answers, they like to discuss their processes.

RE: Why?

AT2: Because it has not been encourage a lot [to only provide an answer]. . . Because this is the answer, we don't know how we got it. . . They have learned that the answer is what is most important [in the past], which is not good.

### Providing Real-Life Activities

The third way to encourage students is to provide real-life activities for students.

American teachers believe that all students can be encouraged by giving real-life connections with math curriculum. Over 90% of the American teachers write about teaching episodes closely related to real world problems. For instance, one teacher uses “The men’s basketball NCAA tournament bracket to show how theoretical probability is what should happen, but not always what will happen” (teaching episode 4). Some teachers use games to teach factorizing common factors, which makes their students experience fun. One teacher asks her students to help her design her room by calculating area of the room and the number of the tiles (teaching episode 5). One teacher asks her students to measure “the slope of the stairs” in her school (teaching episode 3). Over half of the American teachers believe these real-life activities help all students understand concepts.

Teachers’ Role—Balance and Variation (80% vs. 20%, see Table 5.5)

The quantitative data show that 80% of American teachers advocates ideas of balance and variation in pedagogical selecting, instead of picking a single well-chosen teaching method. Both interview and teaching episode data show what American teachers refer to as “making the pedagogical balance” is to adopt different teaching methods in their classrooms, to have a lesson reflection, and then to consider possible changes in the next lesson. The first aspect, to adopt different teaching methods in the classroom, is highly recommended by NCTM 2000 Math Standards. American teachers have learned that students possess multiple-intelligence, so it is necessary to use different methods to fit different learning styles. The second aspect, to reflect and change their lessons for the next-time teaching, is partly shared by the constructivists. For instance, social constructivists believe that teachers should serve as facilitators to guide their



students at the Zone of Proximal Development. Therefore, to reflect and to change their lessons are to best fit to their students' understanding.

### Adopting Different Teaching Methods

American teachers adopt teaching methods in different ways. Some teachers use journal writing, others use poems, and the others use different lecture styles to teach students. Some of these styles include: group vs. non-group, lecture vs. practice, regular lesson vs. projects, activity-based vs. non- activity-based, and the like. In a ninety-minute lesson, although American teachers split time into the similar sections— a fifteen-minute warm-up period, ten-minute of homework checking, twenty-minute of lecturing, twenty-minute of guided practice, and a closure— the various teaching strategies are well-embedded in the different sections. They claim that the NCTM has directly influenced their teaching, as AT3 says in her interview: “I get my NCTM magazines all of the time. I only read certain articles. The ideas are very good, where you break it down so they [students] can understand it.”

American teachers also integrate technology into their classrooms to make teaching different. Some use PowerPoint and overhead projector; others use computer learning software to teach certain content. Many of them communicate to their students through internet. For instance, Ford uses website to make math learning easier for her students:

AT5: I use my website, so when I do something in class, if the child did not get the whole concept, they can go on my website and refresh their memory. Also, the parent can go to the site to be able to help their children.

RE: So you have a website that details the lesson plans.

AT5: Well I don't put the lesson plan that I turn into my office but, what I do in my classroom, and if they know it, and it will tell them exactly what we covered, and what we are going to cover. And if a child was absent can see that we did this this and this. If it was a textbook page assignment, they can do it. If I have a PowerPoint or presentation in class, they can also view that as well. They can actually go back; they can reflect on it or go view it.

Lecturing is a part of American teachers' teaching in their classrooms. However, a lecture in American classroom is personalized. American teachers' lectures truly reflect teachers' personalities and their pedagogical knowledge. Some teachers tend to break the concepts into small pieces; some teach their students in a demonstration style; some may use poems in the lecture; and some may integrate a game to make their teaching interesting. The following example is a typical lecture style from AT5's class:

AT5: My lectures aren't really lectures. I will have a premade guided note, and as we go through the PowerPoint they will fill in the information that I feel is important. On the PowerPoint I will say let's try these together, and then they will work them on their own. Then I will walk around to see if they understand them. Then we may have an activity. I don't think it is a normal lecture.

RE: you just take the problem and break them into more pieces and try to make the students understand this concept, don't you?

AT5: The first thing is I would introduce concept itself, and then the pieces of that concept, whether it is a term that they need to understand or a formula, and we will put it into pieces. Then we will go step by step on how to do something. They do it along with me. Then they get practice. They can work with their neighbor, I have tables in my room, and they can talk to their neighbor.

### Having a Lesson Reflection and Re-Teaching

Aside from the above different teaching methods adopted in American classrooms, American teachers have been aware of lesson reflections. Many of them describe how to change their lesson plans the next time they teach. Some teachers reflect and change their lessons for elaboration. These teachers are satisfied with their teaching; however, they strive to make their teaching perfect by adding something the next time they teach. For instance, the following reflection shows that the teacher believes adding calculators and explaining the application of GCF would make her lesson better.

I was pleased with the overall success of this lesson, but if I had to teach this lesson again, I would change two things: 1. I would give the students examples with larger numbers and teach them how to find the GCF by using the calculators, 2. I would give an overview of why factors are so important immediately after I model the listing method. I would present to the students each unit that will be covered throughout the year, and then explain how factors will be used in each unit. (teaching episode 1)

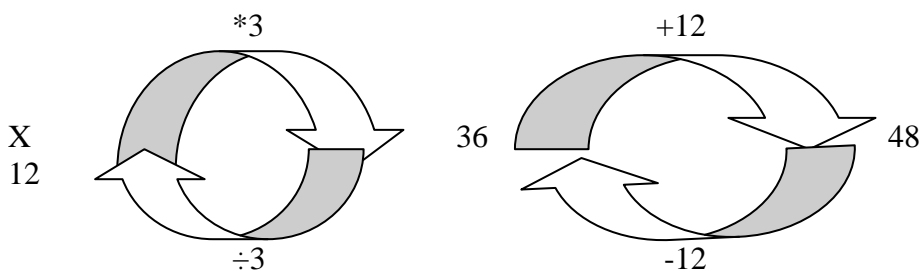
Some American teachers also reflect and change their teaching methods when they fail to make students understand the concepts. The following two teaching episodes describe how these two teachers change their teaching methods to fit their students' needs. The first teacher re-teaches her students by providing real-life examples, while the second teacher re-teaches her students by using her "old method."

On Thursday, we played a review game in preparation for the test on Friday. To my surprise, the students were not doing well on the part of the unit about sets of real numbers. I was very concerned, and decided I needed to do something about the students' lack of understanding of this concept before the test. Consequently, I decided to take 10 minutes before the test and try to explain the concept again. . . Earlier in the week, when we first learned about the different sets of real numbers I had the students create their own Venn diagram. The students struggled initially because all of the Venn diagrams they had seen in the past had circles partially overlap. . . the concept I came up with was how each student could be classified according to teacher, school, and district. If a student was in my class then they were automatically a student in the school, and a student in our district. On the other hand, we could take a student in the district and they would not automatically be a student in our school or in my class. This was similar to real numbers. . . Often times, I feel like I am pulling them [students] in one direction as they pull in the opposite direction. Yet, in this instance I felt like the class and I were pulling in the same direction. (teaching episode 8)

Due to i-lead this year I had to teach unit 8, which is the Algebra unit, very fast. I also use a method of working equations that another teacher was using, it is called the hump method. This is not my normal method of teaching Algebra.

First you tell them to rewrite the problem using order of operation and the humps. Then using the humps to do the opposite of the top hump. So  $48 - 12 = 36$  and  $36 / 3 = 12$

$$3x + 12 = 48$$



Instead of giving an individual test on this unit I gave a group quiz. The results of the group quiz were horrible. The students were due to take their individual test on the whole unit when we returned from the Easter break. I had to modify this plan. I chose to re-teach this unit and to break the test up. When re-teaching one-step equations I used to different methods, my original way of teaching Algebra. First of all I introduced the

vocabulary of: solve, isolate the variable, Addition Property of Equality, Subtraction Property of Equality, inverse operation, Multiplication Property of Equality, and Division Property of Equality. I explain equations were like a scale and the sides had to balance, you could not do something to one side without doing it to the other. I also gave the student steps to use. First, identify the operation on the side of the equation with the variable. I also let the students work class problems in a group so that if they had question they could use peer tutoring to get extra help. Next use the inverse operation to isolate the variable. The original quiz had an average of an F; the new test had an average of a B. (teaching episode 26)

The above two teaching episodes indicate two aspects. On the one hand, the two teachers have been aware of the change of their pedagogies to help students' understanding. On the other hand, these teachers do not adequately envision students' possible misunderstanding of the content when preparing their lessons. They are struggling with adopting new teaching ideas in class. For instance, the first teacher shows her success in this lesson by providing a real-life connection for her students. This lesson also reflects her struggles to put the students and herself in the same directions, as she says at the end of her teaching episode: "Often times, I feel like I am pulling them [students] in one direction as they pull in the opposite direction." The second teacher adopts a "new" method without figuring out the "old" behaviorist teaching beliefs underneath that method. The "new" method facilitates a routine memorization rather than conceptual understanding. In contrast, her "old" method contains more conceptual elements than the "new" one. At least, she has explained that an equation is like a scale, and the sides have to balance in the "old" method. In addition, both teachers claim they do not immediately find their students' misunderstandings when teaching in class. This later feedback implies they are teaching toward demonstrating of the procedures rather than instantly testing students' conceptual understanding.

Some teachers re-teach within a class when they find that their students have difficulty understanding. The new teaching strategies might apply in the re-teach action as described in the following teaching episode:

I had prepared a lesson on Pythagorean Theorem. I thought I had done a good job on presentation of the lesson. I looked out at the students and the students looked at me as if I had just spoken to them in Spanish. The classroom was quiet and none of the student had written a thing on their papers. One of my students finally said, “Ms. Ann, Could you please repeat everything you just said using terms we can understand?” I just laughed and started the lesson again using everyday words that the students used.

### Discussion of Teachers’ Role

Teachers’ role in the values questionnaire contains two cultural elements: teachers’ support for concept learning (item 5) and pedagogical balance (item 11). In the quantitative data, American teachers advocated Western values in item 5 and agree with Eastern values in item 11. In the interview and teaching episode data, we have already analyzed American teachers’ beliefs in depth in these cultural elements. American teachers demonstrate at least three ways toward Western values in item 5: 1) grouping students with different abilities, 2) encouraging students who have low scores, and 3) providing real-life activities. I rarely find American teachers are intentionally providing hints to engage students’ understanding. Most of the teachers believe grouping students with different abilities, either homogenously or heterogeneously, can engage students to participate in the learning activities. They also demonstrate their patience and attention to low achievers. These students are encouraged to speak in class. Most American teachers also believe that real-life activities can help students understand concepts.

The idea of pedagogical balance is manifested in the two ways: different teaching methods adopted for American teachers’ teaching and the reflections to revise their lessons for re-teaching. There is not a dominant teaching method widely accepted by American teachers. The different teaching methods reflect their personalities and their pedagogical knowledge. Some try to make their teaching unique by using poems, journal writing, games, manipulatives, and website-based learning. Some teachers’ lesson reflections or re-teaching reflect their struggle with effective teaching. All these teaching efforts imply that behaviorist teaching is no longer advocated by American teachers, although some teachers use it unconsciously in their classrooms.

American teachers' ideas of pedagogical balance emanated from individualist tradition. When they co-produce their lesson plans, they treat each other equally. As a result, they usually make contribution to the lesson plans equally with emphasizing their own opinions in their own teaching. Each teacher makes his or her own judgments to adopt ideas from others. This tradition helps American teachers develop their unique teaching styles as introduced above. However, this tradition sometimes risks leading to the failure of their teaching. That is, they adopt a teaching idea without deeper understanding of it. When they use this idea in class, they have to re-teach due to the students' misunderstanding (e.g., the 26<sup>th</sup> teaching episode using humps). In contrast, Chinese teachers usually co-produce their lesson plans based on experienced teachers' opinions, an emulative teaching style with less creative and more practical features.

American teachers do not hold the same beliefs as Chinese teachers' regarding one problem with multiple solutions. American teachers believe that one problem with multiple solutions should be adopted in class for the pedagogical balance (e.g., AT6's interview). The reason for adopting this balance is to help students better understand concepts. They believe that students have different learning styles according to multiple intelligence theory. So the different ways to solve a problem will fit the different learning styles. By contrast, Chinese teachers' focus is on the learning tasks, rather than students' thinking styles. Chinese teachers take a multiple-single style, a Taoist tradition, to perceive one problem with multiple solutions. They believe that only the solutions simple and different in nature are valuable to students. So they first encourage students to come up with the different solutions, and then they recommend a few of them to use.

### Learning Process

Concerning the learning process the NCTM 2000/MOE 2001 Math Standards stress that students' learning should be toward understanding, and students should learn math on their own through participating in learning activities. In the indigenous cultural contexts, the learning

process is interpreted as having three cultural elements: 1) learning progression through mental struggle (Eastern) or through sequenced instruction (Western), 2) the means of gaining understanding in learning are either receptive (Eastern) or expressive (Western), 3) the purpose of reviewing in the learning process is either consolidating knowledge or gaining proficiency. In constructivist theory, the learning process is specified as having three aspects: the effectiveness of the lecture, the ontology of conceptual learning, and the origin of motivation. Radical constructivists do not deny the use of lecture for students' understanding. By contrast, social constructivists believe that the lecture is not useful for students' understanding. Ontology of conceptual learning is from individual reflective practice for radical constructivism, in contrast to the acts of communication for social constructivism. Radical constructivists hold that students' motivation comes from the innate drive to understand, while social constructivists believe that students' motivation comes from their participation with others in social-directed activity. The cultural elements the regarding learning process in qualitative data are summarized as follows.

Learning Process—Sequenced Instructions (0 vs. 100%, see Table 5.6)

Quantitative data show that 100% of American teachers advocate that students' learning progression should be made by sequenced instruction, rather than through students' mental struggles. In the interview and teaching episode data, four subthemes are drawn to support this argument: 1) making teaching fun and joyful, 2) emphasizing manipulatives and real-life activities, 3) taking step-by-step instructions, and 4) checking homework.

### Making Teaching Fun and Joyful

American teachers believe that a fun and pleasant atmosphere are very important for learning. They adopt different strategies to achieve this teaching goal, such as making jokes with a very fast class pace (teaching episode 11), asking students to write poems about math for Valentine's Day (AT3's interview), using an animal race (e.g., turtle, bear, and frog) to learning math (teaching episode 2), etc. Some teachers use their body language and poems to make

students feel excited. The following teaching episode illustrates how a teacher designs a story to help students practice GCF after her lecturing:

The students mastered this concept so I decided to give them a “fun” review. Each group was given three task cards. Task 1 required the students to give step-by-step instructions to finding the GCF using words only. Task 2 asked the students to demonstrate how to find the GCF of 60 and 120 using numbers and symbols only. Each group had to act out a scene in Task 3. One of the scenes presented the students with this situation: Shayla (#60) and Jimmy (#120) would like to get married. Shayla’s father rejects the whole idea because he does not feel as though the two have anything in common. The wise Dr. Numbers has invited them to come onto the Jerry Springer show to prove what these two have in common. (teaching episode 1)

This teacher is satisfied with her “fun” review in Task 3. In her three tasks, the first one intends to repeat her previous instructions, which are the basic concepts of GCF. The second task serves as an example to test if students understand the ways to find GCF. The third task is Shayla and Jimmy’s story. Because the focus is on the fun, the teacher simply selects the same numbers, 60 and 120 again, as in Task two. This might lead to a behaviorist teaching direction—when you ask students to solve the same problem, they are reinforced to seek the correct answers, rather than strive to think and compare.

In contrast, AT1’s poem and body performance possess both fun and basic conceptual elements:

For simplifying fractions. . . even over an even. . . you are not through until you divide by two (divisibility rule for two), or a multiple of two, four, six eight, what do we appreciate???? Divisibility Rules. . . This has a rhythm and a beat where I clap twice after I say each number and then move my arms in a circle for what do we appreciate????out in front of my body parallel to the floor before I make the division sign (again with my right arm horizontally positioned in front of my body chest level, when I punch the left hand in a fist over the under the positioned arm) then say the last divisibility at the end “Divisibility rules”. . . then I go into a dance for huh huh yes oh yes, “get it, get it,” says the teacher. . . students say, “got it, got it.” Then the teacher says good. . .(teaching episode 27)

AT1 has used poems with body language to help students memorize the basic concepts for more than twenty years. Her students find fun and this easy to make the lesson memorize. In



the interview, AT1 gives me an example that one of her students still remembered these poems after twenty years. Moreover, this student tries to use these poems to tutor college students.

### Emphasizing Manipulatives and Real-life Activities

Most American teachers believe that a real-life activity or using manipulatives is meaningful for their teaching. The activities are designed based on students' real-life experience.

AT3 presents a typical teaching episode to describe a learning activity in her school:

In an effort to re-enforce the concept of the slope of a line, I decided to have the students in my Algebra I classes measure the slopes of the various stairways at our school. There are four stairways, so I divided each of the classes into four groups with each group being assigned their own stairs. Each group had two people to measure and two people to write down the information that was being gathered. As the teacher, I demonstrated the method that they were to use to get the "rise" and the "run". I moved from group to group, which was not easy because our school is so large and spread-out. Thankfully, I had a motorized scooter and an elevator at my disposal.

The students had to measure different parts of the stairs and compare the results for each landing. They were very actively engaged and really took the task seriously. After returning to the classroom, each group compiled their results. We were shocked to learn that two of the stairways had radically different slopes from the other two. One student remarked, "No wonder I am so tired after going up the stairs by the 7<sup>th</sup> grade hall. I have to lift my legs higher with each step." Each group then made a graph of their findings. The graphs were compared, thus emphasizing the "steepness" of a line.

This activity was monitored by the teacher, but it was truly the work of the students. They were to really see how a mathematical concept is applied in a real-life situation. This also led to discussions of other real-life applications of slope. Teaching the formula was very easy after this activity. The activity also re-enforced measurement skills as well as higher order thinking skills. Cooperative learning was obvious because each student had a task and each contributed to the group results. This was a very successful teaching episode because it allowed the students to truly interact with each other. It fostered intelligent discussion among the students and encouraged social stimulus for those students who otherwise might not have taken an active part in the lesson. (Teaching episode 3)

In the above teaching episode, AT3 first gives students basic concepts of the "rise" and the "run", and then students are asked to measure the stairways in their school. After students finish data collection, they have group discussion to compare the different results. They are also asked to compare the graphs emphasizing the steepness of a line. AT3 believes that she plays a role as a facilitator to monitor students as they get the data for making comparisons. She

emphasizes cooperative learning, which encourages those students who might not be actively involved in math learning. It seems that the activity is going smoothly: acquiring two basic concepts, going to stairways to measure, recording the data, drawing graphs based on teacher's instruction, and comparing the steepness. It is obvious that AT3 does not predict any misconceptions her students might have. Students experience her "fixed instructional sequence" with their surprise: "no wonder I am so tired after going up the stairs by the 7<sup>th</sup> grade hall. I have to lift my legs higher with each step."

Fixed instructional sequence is a normal pattern in American teachers' design of activities. They usually directly give a clear instruction for the procedures and basic concepts that students can follow or operate. Sometimes they use different materials for the same concepts, as AT1 describes in her interview:

AT1: When I did the circumference of a circle, I held my clock; my clock is a circle. I put a Mardi Grad bead around it on a string, and I went around it, and it was short, just by a little bit. And then I showed them how we can go three times across the diameter of the circle. Of course they already knew diameter, and they already knew radius, and when I let it go, all that big long bead was the circumference. So I tell them, say, a "circle fence," circumference. So they watch me. They don't do it. Now, a better way would be if I gave them string to do it. And if I were to give them a tape measure, the activities in curriculum have that for the kids to do. . .

In the above activity, using a string or a tape measure is to examine the conclusion AT1 has already made, rather than to explore new knowledge. AT1 may believe that students experience the different objectives; it would help them understand the relationships between the diameter and the radius of a circle. She strives to make a vivid picture of circumference to her students: a circle fence.

### Taking Step-by-step Instructions

Step-by-step instruction is a common method that American teachers use in their classes. Many teachers believe that this teaching method will help teachers assess students'

understandings. For example, AT5 prefers to use step- by-step instruction in her teaching; she even asks her students to write down these steps:

They tell me that they like how I show them how to do something. I don't just expect them to know how to do something. I take it step by step. . . I believe in working every step out so that the kids understand. If you do not have it written down on paper, you will not be able to see where you made your mistake. I teach them to do that.

Some American teachers believe it is not necessary for students to exactly follow each step that teachers demonstrate on board. The students can skip some of the steps if they understand the concepts, as AT2 says in her interview:

RE: How do you assess your students' understanding?

AT2: When I give them a new process I am a big fan of breaking it down, step by step. So they'll have steps to do the math. So you can tell where they messed up. So they have that concrete step process to do until it becomes internal for them.

RE: So you demonstrate the procedures such as 1, 2, 3, 4, and 5, then your students should follow them, and then you walk around to see if your students understand.

AT2: If they have a problem, I don't make them do it like that, but if they have the wrong answer I might tell them that they missed step three. I don't care if they do it 1, 2, 3. But if they have the wrong answer I try to help them find where they have messed up.

Some American teachers claim that they will combine other effective strategies (e.g., connecting learning materials to songs) to help their students master the steps, as illustrated in teaching episode 25:

My students and I were working on dividing fractions. This concept was very challenging for them to grasp. I explained the textbook steps over and over again. They just could not remember the steps. I was definitely beginning to feel frustrated. I kept thinking, "Why don't they get it?" and "What am I doing wrong?"

Finally, it hit me. These kids could care less about learning this skill. I needed to come up with a creative way to help them remember the steps. As I struggled to come up with a different way, I remembered a song that was currently playing on the radio. The song had lyrics told the story of dividing fractions. The students were in awe that I was able to relate to them on their level. From this day on, which the students hesitated on problems which involved dividing fractions I asked them to recall the song that was discussed in class. (Teaching episode 25)

The teachers who employ a step-by-step instruction in their classrooms usually hold an incremental perspective on students' learning (e.g., AT1's, AT2's, and AT3's interviews). They complain that the current school schedule makes them miss their students. That is, they only have a chance to teach their students every other day. They believe that mathematics should be learned everyday for kids.

AT1: When I was at my other school, we had math every day, same children, so we could cover more and reinforce it more, because I had it every day. You lose children when you skip a day. So if had them on Monday, the next time I see them, is Wednesday. What did they do on Tuesday?

Although American teachers advocate teaching for understanding, a constructivist belief, many of them still emphasize step-by-step teaching with an incremental perspective on learning. This phenomenon could explain the influence of the behaviorist learning culture that American teachers heritage from their indigenous cultural traditions.

### Checking Homework

American teachers usually check students' homework in the bell ringer period. Some of them pick up either even or odd numbers of the problems for lecturing or students' board working, while others may ask students to raise their hands if they have questions. If a student asks a question, they usually explain it in front of the class. Although they do not take the same teaching style for homework checking, their purposes for checking the homework seem the same: 1) to make sure if students get the right answers, 2) to help students who are not able to understand the problems. The following dialogue is drawn from a teacher's interview regarding homework checking:

RE: Do you walk around the classroom during the warm-up time?

AT2: No. I call roll and the first person done goes to the board. Someone else gets called on to do number two, and then we go over that.

RE: You call on everybody to see if they get the right answer?

AT2: Yes.

In summary, American teachers tend to give students sequenced instruction instead of making their students have mental struggle in their teaching. Most of them believe that math learning should be fun and joyful. And manipulatives and real-life situations will help students learn in a fun and meaningful way. A step-by-step instruction is embedded in their various teaching activities. Most of them do not write detailed lesson plans. If they prepare lesson plans together, they usually take the equal role. This leads to their personalized lesson development. They check homework by calling roll or asking students to raise their hands if they have questions. They like to explain students' homework questions or ask students to explain in class.

Learning Process—Expressive (20% vs. 80% see Table 5.6)

About 80% of American teachers believe that expressing ideas is more important than receptive ideas for students' understanding concepts. American teachers explore various ways to help students be able to express their ideas in class. For some teachers, one of the purposes for setting up groups is to create opportunities for those students who are not willing to speak in public. AT4 directly states her grouping purpose in teaching episode 1, "Students will be more likely to participate because a small group is less intimidating and much harder to hide in." AT4 also claims that a team captain will organize group discussions:

AT4: Sometimes I'll have a team captain. So they [some students] may not express it, but they may tell their team captain, and the team captain may bring it up or express that idea for them.

Many American teachers claim that students with low achievement do not like to express their ideas in public. Teachers should avoid calling them during the whole class discussions. Nevertheless, American teachers encourage these students to express their ideas in group discussion. In the interviews, AT1 believes that peer pressure is one of the reasons for these students to keep silent during the whole-class discussions. AT3 claims that some students would like to ask others than to ask her.

RE: To express their ideas, you treat them equally, don't you?

AT1: That's a good idea to harp on, too. I'm the middle school where they're twelve or thirteen years old. They don't want to raise their hand and let everyone else know that they don't know. They don't want to appear stupid. There's such peer pressure at this age. So really, a lot of children do not raise their hands to ask for help.

AT3: OK. At this age, they don't like to be labeled according to performance. They are the ones that never raise hands to participate in anything. They don't want their peers to know they don't understand. That is why I have them in groups. They will talk and discuss in the group of four they are in. They don't mind doing in their groups. They would rather ask a member than ask me. That's how middle school students are.

Most American teachers believe that group discussion benefits both high achievers and low achievers. Group leaders play an important role in organizing in-group discussion. Teachers sometimes are involved in a certain group discussion. By this way, expressing ideas serves two functions: 1) helping students understand concepts, and 2) helping teachers test students' understanding. Aside from these two functions, some American teachers also believe that homogenous groups can help students bring new ideas by brainstorming or by the other techniques.

RE: What is the function for group discussion on an open-ended problem?

AT3: I walk around and I don't give them the answer, but I show them how to get it. I let them brainstorm and encourage them to. . . I will throw a question [open-ended] like that in class discussion as well. They like it because of the dialogue. It encourages them because each of them has their opinions about what is going to happen.

Some American teachers claim that reading helps students understand concepts. For instance, AT3 uses this method to ask her students to read the textbook for understanding; AT4 prefers to use journal reading to enhance her students' understanding and good feelings.

RE: Please describe the way that you ask your student express their ideas in class.

AT3: By listening to their peers, sometimes they pay more attention. Every student wants to read. They understand it better when we read it together. I try to incorporate the two together, because you have to read certain math problem. Once they read it they get it better."

RE: What did you do after students' journal writing?

AT4: Well we've had it where students volunteer to read their writing, . . . sometimes the students may want to read it to their peers, so in the event that ask me, "Miss Dunn,

Can I read this to the class?" I allow them to. Because it's something that they feel proud of and they want to share it with the rest of the class.

Learning Process—Consolidating Knowledge versus Gaining Proficiency (40% vs. 56.6% in Table 5.6)

The quantitative data show that American teachers hold a moderate belief regarding consolidating knowledge and gaining proficiency. This result indicates that the behaviorist belief is no longer dominating teachers' thinking. Teachers are starting to realize that the NCTM's slogan, teaching for understanding, is important for students' learning. American teachers have their own ways to help students consolidate knowledge. They intentionally ask students to memorize the rules and to understand basic concepts at the same time, as AT1 does in her class:

AT1's poem: It takes two to make percent, go right . . . Move it left the sign is out of sight . . . it takes two baby, always Two . . . to make percent, go right!!!

To teach the percent sign % is really one hundred 100 where the one is slanting in the middle / with the two zeroes split one on top and one on bottom. Two zeroes in the sign remind you to shift the decimal two places. . . % the percent sign is always written on the right side of the digits for percent. . . 17% Percent means out of 100 parts so that is why you write the denominator of 100 for any percent as a fraction 17/100. (teaching episode 27)

AT1 writes a poem to summarize the procedures that makes the concept easy for her students to remember. Once students forget how to calculate the percent, AT1 immediately asks them to recite the poem with a series questions that test students' basic understanding (e.g., 17% percent means out of 100 parts). AT1 repeatedly uses this method to consolidate students' math knowledge in her class.

Some teachers consolidate math knowledge in their lecturing periods. They teach one problem, and then ask students to do a similar one, and then have a discussion:

AT2: I like to do a lot of examples. I'll work one, have a student work one, then I'll give them one to work at their desk, then we will work them together on the board. There are a lot of interactions there. So I can judge how they are getting it compared to everybody else.

Some teachers complained that middle school students did not master some basic skills in elementary schools. Consequently, they have to consolidate basic math knowledge in middle schools. Teachers believe that lack of basic skills leads to students with fear and low level confidence in math, as described in teaching episode 24:

I think the fear is that if one doesn't have confidence in the basic skills; math will only get harder. With this thought in mind; many of us live what we feel and believe. As an educator, it is necessary that I have a since of my students attitude toward math as well as their foundation of skills in mathematics.

I begin my school year not only setting the stage for what is to come; but also reviewing the basic skills that will be needed for success. When I say basics, I am referring to terminology, decimal operations, fraction operations, solving proportions, along with one and two-step equations. If I see that there is an area of weakness, I will target that particular skill or skills in spiral practices. As one practices any mathematics skill, the how and why is next on my agenda. I use basic questioning techniques on a regular basis in order to build confidence levels of those in need, but also for myself as an assignment of student understanding. I do this because the topic that will be the next building block for all future mathematics courses in Algebra 1. (teaching episode 24)

American teachers do not prepare specific lessons for consolidating knowledge. Many of them believe that students can learn incrementally. They help students prepare their test in class every day.

RE: If students want to participate in the LEAP, it will require some upper-level skills. In this case should teachers go back and review some skills from the textbooks?

AT6: Yea, um, well the LEAP test is what they take in 8<sup>th</sup> grade. I believe that I don't believe in teaching the test; however, everyday in class they should be receiving information that will help them on the test. It is important to me that when they answer a question to me, they answer it in the same way that they would answer it on the LEAP test. I think it is very important that they have that in the classroom. If they feel that struggle, when they get to the LEAP it won't be too bad.

### Discussion of Learning Process

The learning process contains three cultural elements: 1) the ways of learning progression, 2) the ways of understanding in learning, and 3) the purpose of reviewing in the learning process. American teachers overwhelmingly advocate (0 vs. 100% see Table 5.6) that learning progression can be achieved by a sequenced instruction rather than mental struggles, as



the quantitative data show. Four subthemes emerge from the interviews and teaching episodes to further show how American teachers perceive sequenced instruction. Most American teachers believe that math learning should be fun and joyful, instead of mental struggles. They believe that manipulatives and real-life situations will help students learn in a fun and meaningful way. A step-by-step instruction is embedded in their various teaching activities. They also develop their own ways for homework checking.

American teachers' beliefs in quantitative data tend toward expressive rather than receptive (20% vs. 80% see Table 5.6) in the second cultural element—the ways of understanding in learning—in the learning process. The interview and teaching episode data further support this conclusion. Teachers intentionally create opportunities for low achievers to express their ideas in group discussion. They try to explore new teaching methods (e.g., co-teaching method) to make low achievers get help immediately. They also integrate a variety of methods to help students express their ideas in class, and they believe their students are able to get a better understanding of math when expressing their ideas. Most American teachers believe that group discussion benefits both higher achievers and low achievers. Group leaders play an important role in organizing in-group discussion. Teachers sometimes are involved in a certain group discussion.

Nearly half of American teachers (40% vs. 56.6%, see Table 5.6) tend to support Eastern values regarding the purposes for reviewing in the learning process. That is, the purposes for reviewing are both for gaining proficiency and understanding concepts more fully. The interview and teaching episode data, however, show that American teachers have not developed well-used strategies to achieve these purposes. They intentionally ask students to memorize the rules and to understand basic concepts in the same time. They also ask their students to do some follow-up work after their demonstrations or lectures, and they always check students' comprehension.

Step-by-step instruction is very common in American classrooms. American teachers tend to adopt this method in various teaching activities. This teaching method can either go in the behaviorist direction if it is used repeatedly without concerning students' understanding or the constructivist direction if it is used for the purpose of their understanding the concepts. American teachers who employ a step-by-step instruction in their classrooms usually hold an incremental perspective on students' learning. They believe that students should learn math every day in order to perform well—a behaviorist belief regarding learning. Step-by-step instruction also leads to three weaknesses in teaching: 1) lack of sensitivity to the variations in students' understanding, 2) poor capability for conceptual teaching, and 3) orientation toward procedural goals by default.

#### Individual Caring—An Emergent Theme in the USA Data

In the interview and teaching episode data, American teachers demonstrate their enthusiasm to care for low performers. Teachers usually have positive attitudes to answer low achievers' questions, to protect their privacy, and to help them participate in the group learning. As we discussed before, many teachers believe that grouping students can help low achievers feel comfortable to express their ideas. In many cases, low achievers are protected from giving a wrong answer or receiving low grades. For instance, both AT3 and AT1 are concerned with their kids' emotions and dignity:

AT3 interview: Your high achievers always raise their hands. Sometimes I will call on them to ask if they worked the question, and ask what they get. If it is wrong, I will call on someone else. I'll ask two or three others without giving the answer. That way no one will know if they got it right or wrong and later I will work the problem.

RE: After a few minutes will you explain the procedures? Will you ask many to do the same problem?

AT3: Yes, then I will explain the concept. That time, the lower achievement students won't feel so bad, because some average kids may have gotten the wrong answers too and others may have forgotten that they gave the wrong answers. You have to care for these kids' emotions.

RE: So you are concerned with the dignity of the low achievers, correct?

AT1: The dignity? Yes, oh yes. If he comes to the board and gets it wrong, then they tell him “you need to do this.” Or, I don’t let anyone get made fun of, no laughing like you said with dignity.

RE: How do you protect students’ privacy?

AT1: As best I can! Well, as far as privacy, going over to them, what I do when they have their test score, I give them back their test score and I’ll say the As and the Bs, because they get candy, and they get to go put their stars on the wall, and things like that, and if they’re C D or F, then I show them themselves, I don’t – you made a C, you made a D, I don’t do that. So if they don’t hear me say A or B, they know they have C D or F, and then I go over there and show them their number, 76 is the D, they write it down, they bring it home, they show it to their parents, and they come back with a signature, that’s what I do. That’s bonus if they do that. Now report cards, they have to show the signature, but yeah, as far as trying to embarrass somebody, I’m not about that. Because I wouldn’t want somebody to do that to me, I’m very much that way.

The following interview data shows that AT5 pays great attention to low achievers. She believes that co-teaching can help her achieve a goal of individual caring. Her co-teaching means that the two teachers conduct lessons in the same class. As one teacher stands in front of the students, another teacher can walk around back of the class to help low achievers.

AT5: I like to be a team teacher; I don’t want to be teaching by myself. One year we tried having two teachers in the same classroom and we liked it. If I could be in the back while and she is the front and then when she transitions to the back then I can be in the front. We could share my way of teaching with her way of teaching. . . if this child is not up to speed with everybody else, this teacher can come over there and quietly address that question without the whole class having to hear it. We loved it because she could be assisting someone and I can grade that paper and sit with the child and see where the problem is. In this way I think co-teaching is a good thing.

Teachers’ positive attitudes toward helping low achievers reflect in AT2’s and AT6’s interviews. AT2 has a degree in special education. She has had considerable experience in teaching these kids. For AT6, low achievers can get help from other students’ explanations what they didn’t understand in the lecture.

AT2: I’ll go to the lower kids. I have a degree in special education, so I have taught that population. I won’t leave them out because they see it differently. They just don’t understand the process. A lot of times with that extra explanation, they will be with the middle population, so I don’t ever think that they are a waste of my time.

RE: Do you think if a low achiever raises his hand to ask a question, you would give a detailed explanation in front of the whole class?

AT6: Yes.

RE: What if they still don't understand?

AT6: I will ask the students about the problem to get to the underlying theme.

RE: So you indirectly re-explain it again.

AT6: I ask other students to help them explain.

American teachers are good at helping students establish a good relationship indirectly.

The follow teaching episode provides a vivid example for drawing a girl back to the group. This girl is bullied by others in her group. The teacher does not directly persuade other students to stop their behaviors. Rather, she uses the course content to teach her students to understand the importance of keeping all data in data analysis. Outliers cannot be ignored. As the students understand this math principle, they stop bullying this girl.

I had a class one year that consisted of a group of "mean girls." This group was particularly mean and cruel to one girl because she was "different." At this point in the year I was tried coaxing this group to stop and keep the "outsider" from being angry and wanting to fight.

Around this time, in the curriculum, we had to start a unit on data, graphs, and statistics. A set of vocabulary words to be used were clusters, gaps, and outliers. I told my class that a cluster was where most of the data is and an outlier is the 1 or 2 pieces of data that doesn't fit into the trend in the data.

One of my "mean girls" then asked me if they were the cluster and this one girl was the outlier. At this point, I was speechless. I didn't even address it at that time. We then discussed what would happen to our mean, median, mode, and range if we included our outlier in our calculations and what would happen if we left it out. The class realized the importance of including all data.

That same "mean girl" came to me the following year and told me that they included that one "outlier" student from the year before in the group and they no longer pick on others and that she made all the difference in the group.

I find it interesting, my one lesson about data. My students rolled it over onto their personal lives. They realized that it was important to include all data in their math so why shouldn't they include all students into their groups. (Teaching episode 16)

## CHAPTER SIX: CONCLUSIONS AND DISCUSSION

In recent years, comparative researchers have realized that national culture plays a very important role in teaching and learning (Thomas, 1997; Phuong, Terlouw, & Pilot, 2006). However, the ways and the extent of that influence have not been adequately analyzed in comparative education. As a result, comparative researchers have struggled with describing the essence of teaching and learning in their own countries. This is reflected in current cultural debates (e.g., Wong, 2004, Lopez-Real, Mok, Leung, & Marton, 2004). To an extent, dealing with issues of cultural influence regarding teaching and learning has become a bottleneck in comparative education. This bottleneck has also weakened the argument in the current comparative education discourse that the U.S. K-12 education should learn from Eastern countries such as Japan, China, and Singapore based on the large scale international comparative studies (e.g., TIMSS, PISA).

This study aimed to investigate the influence of the indigenous cultures of China and the USA on teaching and learning in those two countries. Specifically, I have examined how the indigenous culture of teaching and learning mediates middle school math teachers' understandings of constructivism. A major motivation for this study was to understand the transportability of educational practices across national cultural boundaries. Teachers participating in this study in both countries were avowed reform oriented teachers, claims later confirmed by the first questionnaire (Reform-Orientation Questionnaire). They were dedicated to implementing math curriculum reforms based on NCTM 2000 Math Standards in the United States or the MOE 2001 Math Standards in China. In order to understand the indigenous cultural influence on reform teachers' teaching and learning, I needed to clarify the controversies within constructivist discourse, the relationships between constructivist theory and the NCTM and MOE

standards documents, and to develop an appropriate cultural model for this study. Consequently, I have adopted both theoretical analysis and empirical methods in this study.

Theoretical analysis helped me establish an appropriate cultural model that is the foundation of this study. To understand cultural influence in comparative education, one must first understand what accounts for national culture and who belongs to that culture (Wong, 2004). Does a national teaching script exist (Lopez-Real, Mok, Leung, & Marton, 2004)? These troublesome issues are the result, in part, of the paucity of appropriate cultural models in comparative education, as reviewed in chapter 2. Theoretical analysis also helped me understand the varied influences of constructivist theory on the NCTM 2000 Math Standards and MOE 2001 Math Standards.

Empirical investigation included (1) thirty participants in each country filling out three questionnaires and writing 2-3 pages teaching episodes, and (2) six participants in each country being interviewed. The first questionnaire (Reform-Orientation Questionnaire, see Appendix A) is an existing questionnaire testing reform beliefs regarding teaching and learning (Ross, McDougall, & Hogaboam-Gray, 2003). The second questionnaire (Teaching-Style Questionnaire, see Appendix B) was developed based on constructivist theory to investigate radical and social constructivist beliefs. The third questionnaire (Values Questionnaire, see Appendix C) was developed for investigating the indigenous cultural values regarding teaching and learning that are based on the new model. The empirical investigation along with the new model helped us understand in what ways and to what extent indigenous culture influence teaching and learning in the U.S. and China. Theoretical analyses and empirical investigation were shaped by four research questions in this study:

1. What are U.S. and Chinese teachers' understandings of constructivism as embedded in the math standards documents from their countries?
2. What aspects of U.S. indigenous culture and Chinese indigenous culture are relevant to learning and teaching?

3. How does the indigenous culture of China and of the U.S. influence teachers' understandings of learning?
4. How does the indigenous culture of China and of the U.S. influence teachers' understandings of teaching?

This chapter briefly answers the four research questions based on the theoretical analysis and empirical investigation. Then I go on to a discussion of the transportability of educational practices from one cultural context to another. Finally, I present the implications and limitations of this study.

### **What Are U.S. and Chinese Teachers' Understandings of Constructivism as Embedded in the Math Standards Documents from Their Countries?**

The first research question was addressed through the first questionnaire (Reform-Oriented Questionnaire) that established the extent to which teachers identify themselves with the reform agenda; through the second questionnaire (Teaching-Style Questionnaire) that examined the teacher's nuanced interpretations of constructivism; and through an analysis of the US and Chinese reform documents to identify the embedded constructivist assumptions.

The responses to the Reform-Oriented Questionnaire indicate that most of the participants in both countries are reform-oriented. This means most teachers in this study agreed with some basic constructivist tenets reflected in NCTM 2000 or MOE 2001 Math Standards. For instance, teachers should serve as facilitators and co-participants in their classrooms. Learning tasks should reflect students' prior experience (e.g., real-life problems, manipulatives). Students should actively construct math knowledge through communication.

Teachers' understandings of constructivism in this study were further investigated through the Teaching-Style Questionnaire. Teachers' responses to this questionnaire help us understand more fully their constructivist beliefs (radical constructivism or social

constructivism). The overall responses to this ten-item questionnaire demonstrated a systematic differences between Chinese and American teachers with respect to the leaning toward radical and social versions of constructivism, Chinese teachers tended to hold strongly a radical constructivist perspective regarding the source of tasks (item 5). Chinese teachers also held very strong social constructivist beliefs regarding using multiple solutions for the same problem (item 9) and the design of tasks (item 6). The American counterparts held very strong social constructivist beliefs regarding knowledge production (item 1) and the goal of learning (2). The American counterparts also held very strong radical constructivist beliefs regarding the source of tasks (item 5) and design of tasks (item 6). Detailed descriptions can be found in chapter four section one.

### What Aspects of U.S. Indigenous Culture and Chinese Indigenous Culture Are Relevant to Learning and Teaching?

The second research question was addressed through analysis of traditional cultural contexts in China and the USA, through sociological theory (e.g., emergence theory), and through current cultural studies regarding teaching and learning in comparative education. The second research question led to a new cultural model. This new model was the basis for the values questionnaire developed for this study. The empirical data generally confirmed that the cultural model captures aspects of teacher's culturally specific systems of belief in the two countries. I first present the new model, and then I briefly introduce the empirical results of the values questionnaire.

In sociology, researchers have developed a variety of theories and concepts to analyze the relationships between social properties and individual beliefs. One key concept is supervenience which means "if two events are identical with respect to their descriptions at the lower level, then they cannot differ at the higher level" (Sawyer, 2001, p. 556). In this dissertation study, I have adopted the construct of emergence from sociology to help resolve whether a social property can



be reduced to individual properties, and whether the sum of the individual properties constitutes a social property. Emergence theory provides a non-reductive materialist perspective that holds that “social properties are supervenient on individual properties and yet not reducible to those properties” (Sawyer, 2001, p. 580).

From this perspective, I perceive indigenous culture as a collection of interconnected social properties realized in the collective activity of a cultural group. It is dynamically stable but subject to gradual change. Beliefs and values about knowledge are an important part of the indigenous culture of teaching and learning that are realized in the activities concerning schooling within a culture.

The following figure illustrates the relationships of cultural elements regarding the indigenous culture of learning and teaching. As a collection of interconnected social properties, culture is manifested as both cultural precepts or principles and cultural practices as shown below. Mutual influence constitutes the dynamic features of culture, as illustrated in Figure 6.1.

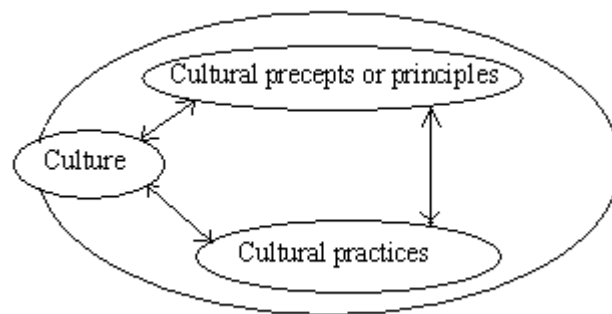


Figure 6.1: Culture and Practice

From the emergentist perspective, cultural precepts and principles are perceived as more stable social properties than cultural practices. Cultural practices are understood as current individual actual practices within a cultural context. This framework implies that one cannot explain individual actual practices completely based on cultural precepts or principles, nor can one fully deduce cultural precepts or principles from examination of individual practice; though

the levels of analysis are deeply interdependent. This conception helps resolve conflicting perspectives in comparative research (see chapter three for details).

In this study, I have applied emergence theory to establish indigenous cultural models in China and the USA. Confucianism and Taoism emerge as the most influential beliefs and values in terms of teaching and learning in China, in contrast with Behaviorism and Individualism in the USA. According to the cultural conceptions in Figure 6-1, I am concerned with the two level interactions, Precepts or Principles and Cultural Practices, when selecting cultural elements in both countries. That is, I first trace back to the histories in both China and the USA in order to find prominent theories, beliefs, and values that have had a long term influence in education. And then I examine if these theories, beliefs, and values are still reflected in current cultural practices in terms of teaching and learning. In addition to resolving troublesome issues in comparative education, this cultural model also helps us examine the transportability of cultural practices across nation boundaries.

A seventeen-item values questionnaire was developed based on this new cultural model. In this questionnaire the statements on the left side reflect Confucian or Taoist perspectives, whereas the statements on the right side stand for Individualist or Behaviorist Perspectives. For instance, item 5 deals with teachers' support for concept learning. On the left side, the statement is "A teacher helps best by providing hints when a student struggles with a problem," in contrast to "A teacher helps best by reframing the problem when a student struggles with a problem" on the right side. The overall responses from participants in this study reflected that the teachers tended to advocate their indigenous beliefs excepting a few items (See Table 5-3, items 11, 17 for American participants; items 2, 6, 8 for Chinese participants). This to some extent provides a validation of the cultural models. In addition, the responses from participants (see Table 5-3) also showed that Chinese teachers demonstrate very strong Eastern beliefs in five of the ten dimensions: Structure of knowledge, teachers' role, learning process, moral outlook, and attitude

relations. Likewise, American teachers hold very strong Western beliefs in the following dimensions: Relation of hierarchy, Learning process, Structure of knowledge, and Value of knowledge. I specifically selected five cultural elements in the dimensions of the teachers' role and the learning process in the values questionnaire as predetermined themes for the further analysis. And these analyses become the main parts to answer the research questions 3 and 4.

### How Does the Indigenous Culture of China and of the U.S. Influence Teachers' Understandings of Learning?

This section intends to answer research question 3. This research question was addressed through interview and teaching episode data. The results in Table 6.1 from the Values Questionnaire reflect Chinese and American teachers' values in the learning process. The learning process items comprise three societal-level cultural elements: 1) the ways learning progresses (item 9); 2) the ways of understanding in learning as either receptive or expressive (item 10); and 3) the purpose of reviewing in the learning process (item 14). Chinese teachers' responses favored Eastern values in all three cultural elements. American teachers' responses however reflected Western values in the first two cultural elements, but fell in the middle position for the third cultural element. Chinese teachers highlighted such ideas as mental struggle, receptive learning, and knowledge consolidation. But American teachers highlighted the ideas of sequenced instruction and expressive understanding. Nearly half of American teachers advocated the Eastern value of knowledge consolidation (item 14).

Chinese teachers believed that students should learn through mental struggle. In the interview data, Chinese teachers also emphasized that mental struggle was a very important aspect for students' math learning. Mental struggle can happen in class as well as outside of class. Many teachers in the interviews expressed a perspective on math learning which was closely related to mental struggle. They also believed that teachers should experience mental struggle first when preparing their lesson plans in order to make students experience mental

struggle in class. In addition to designing tasks for students' mental struggle in class, Chinese teachers also intentionally designed homework problems for their students' long term mental struggle.

Table 6.1: Comparison of Accumulated Percentages on Learning Process

		Eastern values	Western values
Item 9			
Mental struggle	China	60%	33.3%
Sequenced instruction	USA	0	100%
Item 10			
receptive learning	China	86.7%	10%
expressive learning	USA	20.0%	80%
Item 14			
knowledge consolidation	China	96.7%	3.3%
gaining proficiency	USA	40%	56.6%

Note: the missing data are not reported in this table

Yet as the quantitative data show, American teachers overwhelmingly (100% of the sample) advocate that learning progress can be achieved by carefully sequenced instruction rather than mental struggles. Four subthemes emerged from the interviews and teaching episodes showed furthermore how American teachers perceive sequenced instruction. Most American teachers believed that math learning should be fun and joyful, not a mental struggle. They believed that manipulatives and real-life situations would help students learn in a fun and meaningful way. Step-by-step instruction was embedded in their various teaching activities. They also developed their own ways for checking homework. According to interview data (e.g., teacher AT5's interview), Step-by-step (procedural) instruction was very common in American classrooms.

Chinese teachers held that being receptive to the text and the teacher was the primary path toward understanding (item 10). Some openly questioned the ideas of cooperative learning.

They preferred to use traditional Chinese teaching methods (e.g., lecture plus practice) in their classrooms. Others claimed that they strongly agreed with reform ideas such as students' participation in the learning activity. Their interpretations of their concerns and their classroom teaching implied however a strong Chinese traditional belief in their reform teaching. According to teaching episodes 1 and 6, American teachers instead value intentionally creating opportunities for low achievers to express their ideas in group discussion. They explore new teaching methods (e.g., teacher AT5's co-teaching method in the interview) to ensure low achievers receive help immediately. They also value integrating a variety of methods to help students express their ideas in class, and they believed their students were able to get a better understanding of math when expressing their ideas. Most American teachers believed that group discussion benefited both higher achievers and lower achievers. Group leaders played an important role in organizing group discussion. American teachers were involved sometimes in a particular group discussion.

Chinese teachers believed strongly that reviewing and reflecting on knowledge that students had learned led students to gain new knowledge and understanding. Consolidating math knowledge was mainly manifested in Chinese teachers' review lessons. The review lessons were prepared for the evening practice classes (晚自习) and the reviewing classes for high school entrance examinations. Chinese teachers believed that reviewing math knowledge was very important for students' understanding of math knowledge. This belief also implied that math understanding could not be achieved at one time; this was—a Confucian perspective regarding how to learn knowledge. Chinese teachers interpreted reviewing math knowledge as a purifying process, a process which forced students to think clearly and logically.

Yet, nearly half of American teachers (40% vs. 56.6%) tended to support Eastern values regarding the purposes for reviewing in the learning process. In other words the purposes for

reviewing were both to gain proficiency and to understand concepts more fully. The interview and teaching episode data, however, showed that American teachers have not developed effective strategies for achieving these purposes. For instance, teacher AT1 said in the interview that she intentionally asked students to memorize the rules and to understand basic concepts at the same time. In the interviews AT1 and AT5 said that they asked their students to do some follow-up work after their demonstrations or lectures, and they always checked students' comprehension.

### How Does the Indigenous Culture of China and of the U.S. Influence Teachers' Understandings of Teaching?

This research question was addressed through interview and teaching episode data. The teachers' role in the Values Questionnaire contains two cultural elements: 1) teachers' support for concept learning (item 5), and 2) pedagogical balance (item 11). Chinese teachers' responses regarding teachers' role leaned greatly toward Eastern values. Yet American teachers' responses in teachers' role reflected Western values in item 5 and Eastern values in item 11. The accumulated percentages on teachers' role were as follows:

Table 6.2: Comparison of Accumulated Percentages on Teachers' Role

		Eastern values	Western values
Item 5 providing hints encouraging students	China	70%	30%
	USA	13.8%	86.7%
Item 11 balance and variation a single well-chosen method	China	96.6%	0
	USA	83.3%	16.7%

Note: the missing data are not reported in this table.

These comparative results indicated that Chinese teachers display Eastern values (70%), whereas American teachers displayed Western values (86.7%) in item 5. Chinese teachers believed that providing hints was more valuable than encouraging students or reframing tasks. In contrast, American teachers tended to hold the opposite belief.

The interview and teaching episode data further revealed the concrete meaning of this cultural element (item 5). For Chinese teachers, the purpose of providing hints was to reduce the level of difficulty in math learning. Through this method, students were able to engage the learning tasks with a moderate level of mental struggle—a Confucian belief about learning in the Chinese learning tradition. In the interview and teaching episode data, American teachers demonstrated at least three ways to accomplish their goal to encourage students or reframe tasks instead of providing hints: 1) grouping students, 2) encouraging students who have low scores, and 3) providing real-life activities. Most American teachers also believed that real-life activities could help students understand concepts.

Table 6.2 showed that both Chinese and American teachers' responses in pedagogical balance (item 11) reflected Eastern values, a belief that variation and balance should be considered for pedagogical selection. However, their interpretations reflected different emphases. In this study there were four ways Chinese teachers achieve balance and variation: 1) the balanced perspective between open-ended problems and closed problems, 2) the balanced perspective between one problem with multiple solutions and one problem with a single solution, 3) the balance between manipulatives and non-manipulatives, and 4) the balance between conceptual understanding and skill acquisition. In contrast to their Chinese counterparts American teachers interpret the idea of pedagogical balance in two ways: 1) adopting varied teaching methods, and 2) reflecting on their lesson implementation in order to revise their lessons for re-teaching. There was not a dominant teaching method widely accepted by American teachers. The different teaching methods reflected their personalities and their pedagogical knowledge. Some tried to make their teaching unique by using poems, journal writing, games, manipulatives, and website-based learning. Some teachers' lesson reflections or lesson re-teaching reflected their struggle with effective teaching. Chinese teachers generally emphasized tasks and concepts with skills, whereas American teachers emphasized the different

teaching strategies and teaching reflections in this cultural element (item 11, pedagogical balance).

The cause of this value switch in item 11 by American teachers might be tentatively explained. The first way, adopting different teaching methods in the classroom, was highly recommended by NCTM 2000 Math Standards. As analyzed in chapter three, NCTM held a balanced view on teachers' role (NCTM 2000 Standards, p. 18). American teachers moreover may also be influenced by multiple-intelligence theory (e.g., AT6 interview). Students possess multiple intelligence, so it is necessary to use different methods to fit different learning styles. The second way, reflecting on and changing their lessons for re-teaching, was shared partly by the constructivists. Social constructivists, for instance, believed that teachers should serve as facilitators to guide their students at the Zone of Proximal Development. Therefore, reflecting upon and changing their lessons best fitted their students' understanding. Nevertheless, American teachers' teaching efforts implied that behaviorist teaching was no longer advocated by them, although some teachers used it unconsciously in their classrooms.

Chinese teachers' emphasis on pedagogical balance reflected a traditional Chinese belief in Taoism. Taoists believe that open cannot be separated from closed, and multiple cannot be separated from single. In the dynamic process open should go toward closed, and the multiple should go toward the single and vice versa. In accordance with Taoist beliefs, Chinese teachers did not believe the more solutions a problem has, the more value that problem possesses. As introduced in chapter 2 section 2, both NCTM 2000 and MOE 2001 Math Standards highlighted open-ended problem and problems with multiple solutions. The interview and teaching episode data in this study indicated that Chinese teachers have not entirely accepted reform beliefs from MOE 2001 Math Standards regarding task selection.

American teachers believed that one problem with multiple solutions should be adopted in class for pedagogical balance, and different solutions for the same problem would help



students understand concepts in their own ways (e.g., AT6's interview). They believed that students have different learning styles according to multiple-intelligence theory. So the different ways to solve a problem would fit the different learning styles. But Chinese teachers focused on the learning tasks, rather than students' thinking styles. Chinese teachers took a multiple-single approach to perceive one problem with multiple solutions, and an open-closed approach to perceive open-ended problems.

### How Two Emergent Cultural Elements Mediate Teachers' Reform Beliefs

Two cultural elements (emulative teaching and individual caring) emerged from interview and teaching episode data. These individual perceptions fit in with Confucianism and Individualism respectively, though they were not anticipated in the design of the instruments. Emulating model persons is rooted in the Confucian tradition. One of the core beliefs in Analects was the idea of emulation. Confucius defined "Junzi" as an ideal person who should be emulated by others. To care each student was compatible with individualist beliefs (e.g., attribution style, Table 3.3).

#### Emulative Teaching in Chinese Math Education

Emulative teaching refers to a teacher's teaching that was based on model teachers or experienced teachers. Emulative teaching was manifest in both interview and teaching episode data. All Chinese teachers in the interviews claimed that experienced teachers' teaching was very important for new teachers to develop their teaching skills. For instance, CT6 was able to state exactly his mentor teachers' lessons when recalling how his mentor teacher prepared demonstration lessons. Teacher CT5 and CT4 described how teachers in some schools used the same lesson plans to teach. These lesson plans came from experienced teachers. Teacher CT5 and CT4 strongly emphasized the role of research lessons and research groups in teachers' professional development. These lessons and group work reflected well an emulative style.

Emulative teaching had both positive and negative effects. On one hand, emulating an experienced teacher might quickly help new teachers acquire pedagogical knowledge. On the other hand, a teacher might develop a routine and fixed teaching format without any real understanding of the experienced teachers' lessons. For instance, I presented in chapter four a teaching episode (teaching episode 17) following a fixed teaching format in China. There were no unexpected teaching events or real reflections in the teaching episodes. Rather, for this teacher, effective teaching meant to develop standard lesson plans. Teaching episode 17 superficially demonstrated a "sophisticated" way to develop lessons. It implied however a lack of understanding in effective teaching. This emulation as a result becomes a routine in developing his lesson plans.

#### Individual Caring in American Math Education

In the interview and teaching episode data, American teachers demonstrated their passionate care for low performers. Teachers usually displayed positive attitudes toward answering questions from low achievers, protecting their privacy, and helping them participate in group learning. As discussed before, many teachers believed that grouping students could help low achievers feel comfortable to express their ideas. Low achievers were protected in many cases from giving a wrong answer or receiving low grades.

American teachers also strived to explore new teaching methods to help their students experience face-to-face guidance. For instance, AT5 believed that co-teaching could help her achieve a goal of individual caring. Her co-teaching meant that the two teachers conduct lessons in the same class. As one teacher stood in front of the students, another teacher walked around the back of the class to help low achievers and others.

American teachers were good at helping students establish a good relationship indirectly. In chapter four a teacher adopted statistical meanings to stop some students bullying a girl. This girl was bullied by others in her group. The teacher did not directly persuade other students to

stop their behaviors. Rather, she used the course content to teach her students how to understand the importance of keeping all data in data analysis. Outliers cannot be ignored. As the students understood this math principle, they accepted that girl in their group.

### Transportability of Learning and Teaching Assumptions across Cultural Boundaries

Both Individualism and Behaviorism have a big influence on American education as analyzed in chapter three. Behaviorism, however, has not dominated US mathematics education reform since the 1970s. Behaviorist teaching was criticized in Erlwanger's (1973) famous study; and NCTM's series documents (1980, 1989, 1991, 1995, 2000) have presented points of view opposed to Behaviorism. This study showed that American teachers still use behaviorist strategies implicitly (e.g., step-by-step instruction) in their classrooms. This result is consistent with Stigler and Hiebert's findings in the TIMSS study (1999). This study further confirms that even for the reform teachers who advocated NCTM's teaching beliefs, behaviorist beliefs are still implicitly embedded in their teaching strategies. Nevertheless, as different cultural elements compete with each other, behaviorist beliefs become weaker.

Confucianism and Taoism have influenced China's society for more than two thousand years as analyzed in chapter three. This study showed that Chinese teachers did not overtly claim their teachings were guided by these traditional beliefs. They interpreted their teachings based on MOE 2001 Math Standards. Chinese teachers had some struggles when they adopted reform teaching strategies; their traditional beliefs impeded the acceptance of beliefs from MOE 2001 Math Standards. Unlike Behaviorism, Confucian and Taoist influences have not been subject to criticism in recent years in China. The following paragraph will discuss the transportability of the cultural elements in the above four cultural traditions.

Transportability of educational practices refers to the ease of adoption of an idea or method from one culture to another. In this study, I introduce an empirical measure of

transportability; the ease of adoption of a cultural practice from Culture A to Culture B is indexed by the percentage of teachers in Culture B that already hold the Culture A practice/perspective. There is an obvious empirical sense in which this proposal has validity. If  $n\%$  of teachers in Culture B already subscribe to the practice or perspective of Culture A, then the project of transportability only needs to extend to the remaining  $100-n\%$  of teachers. More importantly, though, the percentage of teachers holding to a particular belief or practice is an indicator of the extent to which the belief or practice is familiar to the educational community. Transportability is a multifaceted process. There may be many factors that influence whether a practice or perspective can be adopted. But surely familiarity is one of them. Ideas and practices that are not represented in the current discourse of a community are likely to be more difficult to engage with, initially.

It is an empirical question as to whether this measure will prove useful for educational theorists, administrators, and policy advisors. It will require many efforts to use and apply this measure to discover if the exercise is helpful for predicting problems and promoting educational change. In the meantime, I introduce this measure of transportability because of its a priori plausibility, and as a first step in the long empirical process of validation.

#### Transportability of Cultural Elements in the Teachers' Role

In this section, I discuss the transportability of the two cultural elements in the teachers' role. The evidence indicated that teachers in this study gravitated toward their own indigenous cultural values regarding the first cultural element in teachers' role; Chinese teachers tended to advocate providing hints, whereas American teachers favored encouraging students or reframe tasks. Teachers' indigenous cultural values aside, both NCTM 2000 and MOE 2001 Math Standards emphasize encouraging students. Therefore the Western value of encouraging students might be easier to transport to Eastern culture than to transport the Eastern value of providing hints to Western culture. The quantitative data in this study also supported this conclusion.

Seventy percent of Chinese teachers advocated providing hints, but 30% of Chinese teachers supported the Western belief that favored encouraging students. Yet 86% of American teachers advocated encouraging students, whereas only 13.8% of American teachers agreed with the Eastern belief that preferred providing hints. Another factor mitigating against American teachers providing hints was this: providing hints usually came along with other Eastern cultural elements such as having mental struggle. We could not expect both teachers and students in Western countries to acquire these cultural elements within a short time.

But for the second cultural element in teachers' role (making pedagogical balance and variation) the situation was different. Chinese teachers overwhelmingly advocated the Eastern value (note: 0% advocates the Western value). And 83.3% of American teachers also favored the Eastern value (making pedagogical balance and variation). The Western value in the second cultural element emanated from behaviorist teaching. Many researchers (e.g., Stigler & Hiebert, 1999) claimed that behaviorist teaching still dominated in American classrooms. This study indicated that behaviorist teaching began to lose its power in Western education stage after NCTM's efforts. Behaviorism became an implicit belief of American teachers (see learning process, step-by-step instruction).

American teachers' responses toward Eastern values did not mean they shared the identical beliefs on this matter with Chinese teachers. American teachers instead emphasized adopting different teaching strategies and having teaching reflection for re-teaching. These perspectives and strategies have been recommended by NCTM 2000 Math Standards and other NCTM-based professional activities. In this study, Taoist beliefs regarding balance and variation merged from Chinese teachers' responses. They focused on the tasks (e.g., open-ended problems, one problem with multiple solutions). It was possible to adopt Taoist perspectives for American teachers although they held different perspectives regarding the use of these tasks.

### Transportability of Cultural Elements in the Teachers' View of Learning Process

There was a cultural transition stage in Chinese math education. More specifically MOE 2001 Math Standards did not clearly value the Eastern indigenous cultural elements such as mental struggle and receptive understanding. MOE standards instead strongly emphasized expressive learning (e.g., students should participate in the activity-based learning). This posed a big challenge for Chinese teachers being asked to accept some beliefs in MOE 2001 Math Standards if they already held indigenous beliefs regarding learning through mental struggle (item 9) and receptive understanding (item 10). Holding beliefs regarding mental struggle and receptive understanding became to some extent an obstacle for Chinese teachers in adopting the reform idea of expressive learning from Western culture. Adopting an imported cultural element was a long-time assimilating process. Chinese teachers cannot achieve this in a short time. They did not possess the dispositions toward students' discussions nor the sophisticated skills to shape students' activities.

### Transportability of Chinese Cultural Elements into the U.S. Context

This study suggests that it would be more difficult for American teachers to accept learning through mental struggle. Not one teacher selected this item in quantitative data. It might be easier for American teachers to acquire the Eastern cultural element of receptive understanding than mental struggle. 20% of the American teachers in this study advocated receptive understanding in quantitative data. One of the six American interviewees also stated that students understand concepts through receptive learning (AT6's interview).

American teachers have been in the cultural transition stage regarding the purposes for reviewing in the learning process (item 14). Nearly half of them (40%) advocated that reviewing was in order to consolidate knowledge rather than gain proficiency. But of the three cultural elements (the ways of learning progression in item 9, the ways of understanding in learning in item 10, and the purpose of reviewing in the learning process in item 14) the Eastern perspective

on consolidating knowledge was probably the one that was easiest to transport to Western culture.

There were several reasons for the different responses in the above three cultural elements (item 9, item 10, and item 14). First, almost half of the American teachers advocated Eastern values in item 14 resulted from the NCTM's influence. NCTM 2000 Math Standards clearly stated that teaching was for students' understanding. But NCTM 2000 Math Standards did not strongly recommend gaining proficiency. Gaining proficiency was associated to some extent with the behaviorist teaching beliefs. That over half of American teachers chose this cultural element indicated that behaviorist beliefs still influenced American teaching. Second, receptive understanding was not recommended by NCTM 2000 Math Standards. That 20% of the American teachers accepted this belief revealed international influence (e.g., globalization, and international comparative studies) on teachers' beliefs. For instance, TIMSS results showed that students in Asian countries or regions such as Japan, Singapore, Taiwan, and Hong Kong outperformed the USA counterparts. Math educators and teachers considered utilizing some Eastern teaching strategies in the Western classroom.

### Implications

This study can help educators and math teachers precisely understand to what extent they can learn teaching strategies from other cultures. For instance, the transportability of cultural elements discussed above indicates Eastern values of mental struggle and receptive learning did not fit Western teaching and learning. American teachers might not acquire such cultural elements in a short time. It might be easier for them to acquire a Taoist belief (e.g., balance and variation) than to acquire Confucian beliefs regarding teaching and learning. Math teachers and educators in America might also apply Taoist beliefs for design of the tasks (e.g., Eastern ways to see open-ended problems, one problem with multiple solutions). On the other hands, Chinese teachers might begin to learn how American teachers care about low achievers, which might be

easier for them than to acquire group discussion that American teachers usually applied sophisticatedly in their classrooms.

This study provided a new approach to investigate teachers' reform beliefs, an approach using Reform-Orientation Questionnaire to test teachers' general reform beliefs and then using Teaching-Style Questionnaire to test their radical or social constructivist beliefs. The second investigation of teachers' beliefs, either radical or social constructivism, is very important and may contribute to future research. On the one hand, most of the current studies (e.g., Ross, McDougall, & Hogaboam-Gray, 2003) on testing the reform beliefs merely focused on the tenets either from NCTM documents or from the heterogeneous constructivist discourse. And it is not adequate to understand teachers' constructivist beliefs of teaching and learning. For instance, advocating the use of open-ended problems in teaching is usually identified as reform beliefs in the NCTM-based assessment (e.g., Reform-Orientation Questionnaire in this study). However, advocating the use of open-ended problems in teaching does not guarantee a reform belief or reform teaching. Teachers who use this approach can either go to a radical constructivist orientation, or go to a social constructivist orientation, or even go to a behaviorist orientation (Huang, 2009). On the other hand, figuring out teachers' reform beliefs (radical or social constructivism) can help us understand how these beliefs are along with the teaching goals. The disconnections between reform beliefs and teaching goals were perceived as one of the limitations in NCTM's statements of constructivist beliefs.

The results from this study might benefit policy makers and researchers to reflect math standards in their own cultures. This study indicates that math standards' writers should consider the indigenous cultural influence on teaching and learning in their own countries. Quickly adopting cultural elements regarding teaching and learning from outside of their culture might result in teachers' struggles (e.g., Chinese MOE math standards). Meanwhile, quickly adopting a



theory (e.g., constructivism) as theoretical underpinning of the math standards might limit to stress principles of teaching and learning, as I analyzed in chapter four section one.

### Limitations and Recommendations

The first limitation is that this study only portrays specific cultural influence (seven cultural elements) in China and the USA. Further investigations of other cultural elements, either societal-level or individual-level, would be valuable for understandings of the reform teaching in both countries. Indeed, Only Confucianism and Taoism were selected in the new cultural model in this study. Chinese Buddhism was not included in the new model for the investigation. Given that Buddhism is also important in Chinese culture, future research might consider adopting Buddhism in the new cultural model.

Second, the sample of this study is middle school teachers. How indigenous cultural elements mediate elementary and high school teachers' understandings of constructivism is not investigated.

Future studies might focus on the following: 1) investigating how the other cultural elements that are not fully investigated in this study in the new cultural model influence teachers' reform beliefs, 2) investigating different samples (e.g., elementary school teachers, high school teachers), 3) investigating the transportability of the certain cultural elements from one cultural context to another one (e.g., knowledge consolidation, attitude relations), 4) revising the new cultural model (e.g., incorporating Buddhism in it) for new investigations.

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## Appendix A: Reform Orientation Survey

Explanations: Mark a number following each statement that best fits your opinion. 1 represents strongly disagree; 2 means somewhat disagree; 3 represents neutral, 4 stands for somewhat agree; and 5 is strongly agree.

	strongly disagree					strongly agree				
	1	2	3	4	5	1	2	3	4	5
1 I like to use math problems that can be solved in many different ways										
2 I regularly have my students work through real-life math problems That are of interest to them	1	2	3	4	5					
3 When two students solve the same math problem correctly using two different strategies, I have them share the steps they went through with each other	1	2	3	4	5					
4 I tend to integrate multiple strands of mathematics within a single unit	1	2	3	4	5					
5 I often learn from my students during math time because my students come up with ingenious ways of solving problems that I have never thought of	1	2	3	4	5					
6 It is not very productive for students to work together during math time	1	2	3	4	5					
7 Every child in my room should feel that mathematics is something he/she can do	1	2	3	4	5					
8 I integrate math assessment into most math activities	1	2	3	4	5					
9 In my classes, students learn math best when they can work together to discover mathematics ideas	1	2	3	4	5					

10	I encourage students to use manipulatives to explain their mathematical ideas to other students	1	2	3	4	5
11	When students are working on math problems, I put more emphasis on getting the correct answer than on the process followed	1	2	3	4	5
12	Creating rubrics for math is a worthwhile assessment strategy	1	2	3	4	5
13	In my class it is just as important for students to learn data management and probability as it is to learn multiplication facts	1	2	3	4	5
14	I don't necessarily answer students' math questions but rather let them puzzle things out for themselves	1	2	3	4	5
15	A lot of things in math must simply be accepted as true and remembered	1	2	3	4	5
16	I like my students to master basic mathematical operations before they tackle complex problems	1	2	3	4	5
17	I teach students how to explain their mathematical ideas	1	2	3	4	5
18	Using computers to solve math problems distracts students from learning basic math skills	1	2	3	4	5
19	If students use calculators they won't master the basic math skills they need to know	1	2	3	4	5
20	You have to study math for a long time before you see how useful it is	1	2	3	4	5

## Appendix B: Teaching Style Questionnaire

Directions: The following items provide alternative interpretations of matters related to learning and teaching of mathematics (responses 1 or 4). Please take a few moments to consider which position most fully captures your own beliefs. Then circle one of these two alternatives, or circle an intermediate number (2 or 3) that most closely reflects your intermediate level of belief in both alternatives. (Please do not pick a point on the scale between these whole number options.) If your opinion is not captured within these broad alternatives, please circle N/A and provide a brief explanation of your belief.

- |   |   |
|---|---|
| <p>1 Teachers and students actively co-produce knowledge in learning activities</p> <p style="text-align: center;">1                      2                      3                      4</p> <p>N/A: _____</p>   | <p>The teacher's role is to guide students' learning based on their superior understanding of the content</p> <p style="text-align: center;">3                      4</p>                               |
| <p>2 Content understanding is the most important goal for students' learning</p> <p style="text-align: center;">1                      2                      3                      4</p> <p>N/A: _____</p>  | <p>Logical and critical thinking skills are the most important goal for students' learning</p> <p style="text-align: center;">3                      4</p>  |
| <p>3 Facilitating learning means monitoring students' work and engaging with them about discrepancies in their interpretations of the content</p> <p style="text-align: center;">1                      2                      3                      4</p> <p>N/A: _____</p> | <p>Facilitating learning means organizing students' discussions and making sure students' ideas are valued and debated</p> <p style="text-align: center;">3                      4</p>                  |
| <p>4 Lecture is often a useful approach for helping students understand complex ideas</p> <p style="text-align: center;">1                      2                      3                      4</p> <p>N/A: _____</p>   | <p>Lecture generally is not useful for helping students understand complex ideas. Students need to encounter ideas in activity settings</p> <p style="text-align: center;">3                      4</p> |

5	Teachers should provide tasks geared to the content to be learned	1 2	Students should choose or create learning tasks reflecting their interests	3 4
N/A:				
6	The teacher's effort should go to developing a repertoire of challenging tasks that can lead to students' understanding	1 2	Predesigned tasks are okay, but the most effective tasks are designed in the process of teaching	3 4
N/A:				
7	Engaging with open-ended and manipulative tasks is an essential element in a student's mathematical development	1 2	Engaging with open-ended and manipulative tasks is an enjoyable add-on to a sound math curriculum	3 4
N/A:				
8	Conceptual learning is primarily a result of individual reflective practice	1 2	Conceptual learning is shaped in acts of communication	3 4
N/A:				
9	The main purpose of using multiple solutions for the same problem is to help students deeply understand concepts	1 2	The main purpose of using multiple solutions for the same problem is to help students develop flexible thinking habit	3 4
N/A:				
10	When instruction is properly structured, students' motivation comes from the innate drive to understand	1 2	Students' motivation comes from their participation with others in socially-directed activity	3 4
N/A:				

## Appendix C: Values Questionnaire

Directions: The following items provide alternative interpretations of matters related to learning and teaching of mathematics (responses 1 or 4).

Please take a few moments to consider which position most fully captures your own beliefs. Then circle one of these two alternatives, or circle an intermediate number (2 or 3) that most closely reflects your intermediate level of belief in both alternatives. (Please do not pick a point on the scale between these whole number options.) If your opinion is not captured within these broad alternatives, please circle N/A and provide a brief explanation of your belief.

1	Knowledge is an enduring pursuit	Knowledge comes with experience
1	2	3 4
N/A:		
2	Knowledge is to be valued for itself	Knowledge is to be valued for what it enables one to do
1	2	3 4
N/A:		
3	Knowledge is the essence of life	Knowledge can enhance one's life
1	2	3 4
N/A:		
4	Learning is to be valued because it enhance one's social standing	Learning is to be valued because it provides opportunity for material success
1	2	3 4
N/A:		



5	A teacher helps best by providing hints when a student struggles with a problem	A teacher helps best by reframing the problem when a student struggles with a problem
1	2	3 4
N/A:		
6	The person who has reached the pinnacle of their field surely has ideas and methods we should emulate	The person who has reached the pinnacle of their field may have ideas and methods worth considering
1	2	3 4
N/A:		
7	The individual should consider the collective good ahead of their own	The collective good is best served by individuals working in their own self interest
1	2	3 4
N/A:		
8	Motivated students merit more attention from their teacher	Students deserve equal attention from their teacher
1	2	3 4
N/A:		
9	Students learn through mental struggle	With properly sequenced instruction, students can learn incrementally through persistent effort
1	2	3 4
N/A:		
10	Being receptive to the text and teacher is the primary path to understanding	Expressing ideas is necessary for gaining understanding
1	2	3 4
N/A:		

12 What is basic knowledge and what is elaborated knowledge depend on one's perspective at the moment	1                      2                      3                      4	Knowledge start with what is basic and builds upon that base to achieve elaborated knowledge
N/A: _____		
13 Humility is a virtue for student learning	1                      2                      3                      4	Confidence is the foundation for learning
N/A: _____		
14 Reviewing and reflecting on knowledge that students have learned leads students to gain new knowledge and understanding	1                      2                      3                      4	Reviewing and reflecting on knowledge that students have learned leads to greater proficiency with skills
N/A: _____		
15 Assessments should give teachers insight into students' level of understanding	1                      2                      3                      4	Assessments should demonstrate for teachers what tasks students can or cannot do
N/A: _____		
16 People from impoverished background can achieve great learning by diligent and persistent efforts	1                      2                      3                      4	People from impoverished background can achieve great learning if they are given special opportunities and resources to succeed
N/A: _____		
17 Positive and negative coexist in all things	1                      2                      3                      4	People must shun the negative and strive for the positive
N/A: _____		

## Appendix D: Actual Teaching Practice

Recall an actual 10-20 minutes teaching episode in your career in which students really seemed to be having a strong learning experience. Please describe the lesson, highlighting the following aspects: your role as the teacher, student-teacher interactions, and any unexpected events. Conclude with a summary of why you think this teaching episode was successful. (Please be detailed, your response should take **about 30 minutes**, and include about **three pages** of text)

## VITA

Lingqi Meng was born in Dehui, Jilin Province, the People's Republic of China (PRC). After completing high school in his neighborhood, he attended the Northeast Normal University (NENU) at Changchun, Jilin Province. After receiving his bachelor's degree in mathematics in 1987, he became a lecturer at the Educational Institute of Jilin Province. Lingqi pursued his master's degree in mathematics at NENU in 1995 and completed this in 1997. Lingqi was promoted to associate professor in 2001.

In 2004, Lingqi became a doctoral student at Louisiana State University (LSU), majoring in mathematics education program. While working on his doctoral studies, Lingqi acquired a second master's degree in curriculum theory at LSU in 2007. The degree of Doctor of Philosophy will be awarded to Lingqi Meng at the December 2009 commencement ceremony.